There arises a natural question what errors we commit when we replace functions that satisfy some equations only approximately by the exact solutions to those equations. Some tools to evaluate them are provided within the theory of stability of functional equations (nowadays often called Ulam’s type stability), which has been developed in connection with a question posed by S.M. Ulam in 1940 (see [3, 7, 14, 15, 16]). The talk contains some basic definitions and (early and recent) results concerning that stability of functional equations; in particular, the notions of superstability and hyperstability are discussed. The following three definitions describe partially the main ideas of them (see [7]).

**Definition 1.** Let $A$ be a nonempty set, $(X, d)$ be a metric space, $C \subset \mathbb{R}_+^A$ be nonempty, $T$ be an operator mapping $C$ into $\mathbb{R}_+^A$ and $F_1, F_2$ be operators mapping a nonempty set $D \subset X^A$ into $X^{A^n}$. We say that the equation

$$F_1 \varphi(x_1, \ldots, x_n) = F_2 \varphi(x_1, \ldots, x_n)$$

(1) is $(E, T)$–stable provided for any $\varepsilon \in E$ and $\varphi_0 \in D$ with

$$d(F_1 \varphi_0(x_1, \ldots, x_n), F_2 \varphi_0(x_1, \ldots, x_n)) \leq \varepsilon(x_1, \ldots, x_n), \quad x_1, \ldots, x_n \in A,$$

(2) there exists a solution $\varphi \in D$ of equation (1) such that

$$d(\varphi(x), \varphi_0(x)) \leq T \varepsilon(x), \quad x \in A.$$  

(3)

Roughly speaking, $(E, T)$–stability of equation (1) means that every approximate (in the sense of (2)) solution of (1) is always close (in the sense of (3)) to an exact solution of (1).

**Definition 2.** Let $A$ be a nonempty set, $(X, d)$ be a metric space, $\varepsilon \in \mathbb{R}_+^A$ and $F_1, F_2$ be operators mapping a nonempty set $D \subset X^A$ into $X^{A^n}$. We say that equation (1) is $\varepsilon$–hyperstable provided every $\varphi_0 \in D$, satisfying (2), fulfills equation (1).

**Definition 3.** Let $A$ be a nonempty set, $(X, d)$ be a metric space and $F_1, F_2$ be operators mapping a nonempty set $D \subset X^A$ into $X^{A^n}$. We say that operator equation (1) is superstable if every $\varphi \in D$, that is unbounded (i.e., $\sup_{x,y \in A} d(\varphi(x), \varphi(y)) = \infty$) and satisfies

$$\sup_{x_1, \ldots, x_n \in A} d(F_1 \varphi(x_1, \ldots, x_n), F_2 \varphi(x_1, \ldots, x_n)) < \infty,$$

is a solution of equation (1).

The lecture focuses, in particular, on stability of the difference equation of the form

$$x_{n+1} = F(x_n),$$

its generalizations and related functional equations. Also, stability of some conditional equations of the forms

$$g(x + y) = g(x) + g(y), \quad x \perp y,$$

$$g(x + y) = g(x)g(y), \quad x \perp y,$$

are considered (cf. [4, 5, 6]) with some relations $\perp$ patterned on some classical orthogonality notions.

Some examples of stability outcomes for differential and integral equations (cf. [1, 2]) are provided, as well.
References

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Major research interests: functional equations and inequalities with their applications, Ulam’s type stability (e.g., of difference, differential, functional, integral and operator equations), real and functional analysis, fixed point theory.

Author of over 100 papers that are already printed or accepted for publication.

Chairman of the Scientific Committee of the series of conferences: International Conference on Functional Equations and Inequalities (ICFEI) (http://uatacz.up.krakow.pl/icfei/15ICFEI/)


Chairman of the Scientific and Organizing Committees of the conference: Conference on Ulam’s Type Stability, Ustroń (Poland), June 2-6, 2014 (http://cuts.up.krakow.pl/)

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Editor (jointly with Th.M. Rassias) of the monograph Functional Equations in Mathematical Analysis (nearly 750 pages; collection of 47 papers of 67 authors), volume 52 (2013) of Springer Optimization and Its Applications series, dedicated to the 100th anniversary of S.M. Ulam

Lead Editor of Banach Center Publications volume 99 (2013) titled: Recent Developments in Functional Equations and Inequalities. Selected Topics

Lead Guest Editor of Abstract and Applied Analysis annual special issues: Ulam’s Type Stability (http://www.hindawi.com/journals/aaa/type.stability/) in the years 2012, 2013

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