Collocation method – a powerful tool for solving singular ODEs and DAEs

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Abstract: We deal with boundary value problems for systems of ordinary differential equations with singularities. Typically, such problems have the form

$$z'(t) = F(t, z(t)), \quad t \in (0, 1], \quad B_0 z(0) + B_1 z(1) = \beta,$$

where $\lim_{t\to 0} F(t, z(t)) = \infty$ and $\lim_{t\to 0} \partial F(t, z)/\partial z = \infty$. The analysis is usually done for the model equation

$$z'(t) = \frac{1}{t^\alpha} M z(t) + f(t, z(t)), \quad t \in (0, 1], \quad B_0 z(0) + B_1 z(1) = \beta,$$

where $f(t, z)$ may also be in the form of $g(t, z)/t$ with a smooth function $g(t, z)$. For $\alpha = 1$ the problem has a *singularity of the first kind*, while for $\alpha > 1$ the singularity is commonly referred to as *essential singularity*. We briefly recapitulate the analytical properties of the above problems with a special focus on the most general boundary conditions which guarantee their well-posedness.

To compute the numerical approximation for z we use polynomial collocation, because the method retains its hight order even in case of singularities. The usual high-order superconvergence at the mesh points does not hold in general, however, the uniform superconvergence is preserved (up to logarithmic factors). We will discuss how the collocation performs for problems with unsmooth inhomogeneity $g(t, z)/t$ [1].

The updated version [2] of the MATLAB code `bvpsuite1.1` [3] with the special focus on the above problem class has been implemented. For higher efficiency, estimate of the global error and adaptive mesh selection are provided. The code can be applied to arbitrary order problems in implicit form. Also systems of index 1 differential-algebraic equations (DAEs) are in the scope of the code. We present current results obtained for higher index DAEs and illustrate the performance of the software by means of numerical simulation of models in applications.

References:

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[2] Auzinger, G., Kitzhofer, G., Koch O., Pulverer, G., Schöbinger M., Weinmüller, E.B., Wurm, S., `bvpsuite2.0` – a New Version of a MATLAB Solver for Singular Implicit BVPs. *In preparation.*

[3] Kitzhofer, G., Koch O., Pulverer, G., Simon, Ch., Weinmüller, E.B., The New MATLAB Code for the Solution of Singular Implicit BVPs. *JNAIAM J. Numer. Anal. Indust. Appl. Math.*, 5 (2010), 113–134. Available from `http://www.math.tuwien.ac.at/∼ewa`.

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Ewa B. Weinmüller has been a member of the Analysis and Scientific Computing Department at the Vienna University of Technology since 1975. She studied mechanical engineering at the University of Technology in Poznan, Poland, and obtained her MSc degree in 1974. After moving to Austria in 1975 she continued her studies in technical mathematics and physics at the University of Vienna and Vienna University of Technology and completed them in 1979 with a PhD degree advised by Hans J. Stetter. In 1987/88, she was a visiting professor at the Simon Fraser University in Burnaby BC, and in 1996 at the Imperial College in London. Since 2004 she is the coordinator and spokeswoman of the Research Unit "Numerics and Simulation of Differential Equations". Her research interests are analysis and numerical treatment of ordinary differential equations with singularities, especially a posteriori error estimates, defect correction algorithms, mesh adaptivity in the context of boundary value problems in ordinary differential equations and differential-algebraic equations, and software development in Matlab. She is the author and co-author of more than a hundred scientific publications.