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Published by WSEAS Press
www.wseas.org

ISBN: 978-960-474-235-6

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All papers of the present volume were peer reviewed by two independent reviewers. Acceptance was granted when both reviewers' recommendations were positive.
See also: http://www.worldses.org/review/index.html

ISBN: 978-960-474-235-6

World Scientific and Engineering Academy and Society

# Differential Equations On Metric Graph

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# Preface

This is a development report for the investigation of the partial differential equation networks. In this report, we mainly discuss the stabilization of the wave equations with variable coefficients defined on the metric graphs. These networks might be discontinuous or with circuits.

In the past decades, there have been a lot of literature studying the controllability, observability and stabilization for the elastic system. Some nice results and innovative approaches have obtained. For examples, Rolewicz in [95] investigated the controllability of systems of strings; Cox and Zuazua in [22] studied decay rate of the energy of single string system; Xu and Guo in [111] studied the stabilization of a string with interior pointwise control and obtained the Riesz basis property of the eigenfunctions of the system; more recent papers for single string system, we refer to [128], [120] and the reference therein. While single Timoshenko beam treated as two weakly internal-coupled vibrating strings have been studied under various boundary conditions, for instance, see [110], [112], [113],[114] and [115]. Under different control laws, the exponential stabilization and the Riesz basis property of those systems were obtained. There were some nice results for the serially connected strings system, here we refer to literature [15], [62], [68] and [71], in which the authors used the multiplier approach to obtain stabilization for the wave equations by boundary control. In particular, under certain conditions, Liu et al in [71] obtained the exponential stabilization for a long chain of vibrating strings. Guo et al in [43] gave an abstract sufficient condition to deal with Riesz basis generation and apply it to serially connected strings. For other type of serially connected elastic system such as Euler-Bernoulli beams and Timoshenko beams, many authors had made great effort on the control and stabilization of the system, for instance, see [96], [16], [103] and [119].

The study of the differential equations on graphs (or networks) was derived from distinct science background. The questions arise the high-tech such as chip interconnect problem and electron motion in a molecule. The differential equations on graphs was investigated in [87] and [40] for the scattering problem of the free electrons. Since then, there were a great deal papers studying the properties of the differential equations, we refer to two works [90] and [12], in which the authors gave a brief review of results on this aspect. As for spectral problem of the differential equations on the graphs, there were many nice results, we refer to the works of J. von Below, F. Ali Mehmeti and S. Nicaise, please see [10], [1] ,[30] and the references therein. The elastic networks are important class of the differential equations on graphs. As to the modelling and control problem of the elastic networks, we refer to the early works [75] and [102]. More recent development on controllability, observability and stabilization of the network of strings, we refer to a book [34] and a report [129], in where there is a complete list of references on the study of network of strings.

We observe that most of the literature aforementioned mainly deal with the differential equations on graphs with constant coefficients and system continuity, there are a few works discussing the system with variable coefficients and discontinuity. Therefore, we choose the networks with variable coefficients as our research object. Our project includes the following two aspects:

(1) Stabilization of elastic network with variable coefficients;

(2) Identification of the network structure.

In the first aspect, we mainly discuss design of the feedback controllers involving the location of controllers and availability of controllers and stability analysis of the close loop system. In the second part, our attention focuses on identifying the shape of the network structure by measurement. These questions have important application in the real world.

This report is merely a development report for investigation of one dimensional wave networks. The first four chapters are basic materials on the elastic networks. Chapters 5–9 are on the control and stabilization of networks of strings. These works are finished recently. As to the networks of Euler-Bernoulli beams and Timoshenko beams, we will give an investigation report on them in the future. This research is supported by the Natural Science Foundation of China Grant NSFC-60874034 and partially supported by WSEAS.

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