# Numerical simulation of the sound waves interaction with a supersonic boundary layer

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*Abstract:* - In the paper on the basis of the direct numerical simulation the supersonic flow around of the infinitely thin plate which was perturbed by the acoustic wave was investigated. Calculations carried out in the case of small perturbations at the Mach number M=2 and Reynold's numbers Re<600. Two types of acoustic waves were investigated: sliding (φ=0), two dimensional incident waves (χ=0). It is established that the velocity perturbation amplitude within the boundary layer is greater than the amplitude of the external acoustic wave in several times. At the small sliding and incidence angles the velocity perturbations amplitude increased monotonously with Reynold's numbers. At rather great values of these angles there are maxima in dependences of the velocity perturbations amplitude on the Reynold's number. At the fixed Reynolds's number and frequency there are critical values of the sliding and incidence angles at which the disturbances excited by a sound wave are maxima. The oscillations exaltation in the boundary layer by the sound wave more efficiently if the plate is irradiated from above.

*Key-Words:* - supersonic boundary layer, establishing method, acoustic waves, interaction, receptivity, numerical simulations

## 1 Introduction

The questions on the interaction of a supersonic boundary layer with acoustic waves which are considered in the present paper were raised mainly in connection with the problem on the turbulence formation. At present the most complex problem on the prediction of the transition position in the boundary layer flows is related to the receptivity of these flows to the external effects. It appears that this problem was discussed in detail for the first time in [1], and till now many works were carried out on it. However this problem has been studied more thoroughly both experimentally and theoretically for the subsonic flows. A review of the early works of the influence acoustic field effect on the transition from a laminar supersonic boundary layer to the turbulent has been given in [2].

The first attempts to investigate the interaction of the sound waves and supersonic boundary layers on the basis of the stability theory of parallel flows were undertaken in [3,4]. Authors of [5] studied the interaction of sound with a supersonic boundary layer experimentally where the main results of theory [4] were confirmed. Studies [3, 4] have shown that in a result of the interaction of external acoustic waves with the boundary layer the disturbances are excited inside of the boundary layer which amplitudes exceed intensity of the incident sound wave many times. However, as it was noted in [2] these perturbations belong to a continuous spectrum and cannot generate unstable waves (Tollmin-Schlichting waves) in parallel flows. Generating of the growing indignations by an acoustics is possible only in nonparallel flows.

For subsonic flows effective generation of Tollmin-Schlichting waves by a monochromatic acoustic wave is possible only on a strong nonparallelism (inhomogeneity), since the wavelength and phase velocities of these waves are very different from the characteristic oscillations of the boundary layer, which was demonstrated experimentally in [6]. On the basis of numerical integration of the unsteady Navier-Stokes equations [7] it was basically conformed that the conclusion that there is an intense generation of oscillations in a boundary layer by sound only at strong inhomogeneities.

In the case of supersonic velocities the acoustic and hydrodynamic wavelength, and also the corresponding phase velocities can be nearly the same. Therefore the weak flow inhomogeneity is sufficient for their mutual generation. The conditions for the onset of autooscillations and sound generation of the growing indignations in the supersonic shear flows in the jets and mixing layers were studied for the first time in [8]. The advanced approach based on the idea of the possibility of the mutual influence of the acoustic and hydrodynamic waves has been demonstrated in [9].

The problem on the excitation of unstable waves in the supersonic boundary layer by sound was considered theoretically in [10]. Using a controlled acoustic field in [11] it was shown experimentally that the boundary layer receptivity to the acoustic disturbances depends on the location of the interaction region. In particular it was found that the intensity of the hydrodynamic waves generated by the sound reaches its maximum values when the interaction region is located near the leading edge of the model, the lower branch of the neutral stability curve and the "sonic" branch of the neutral stability. The agreement between the theoretical conclusions and the experiments on the acoustic excitation of unstable waves is discussed in [12]. The generation of sound waves by a transitional boundary layer has been revealed experimentally in [13]. Theoretical studies mentioned above were approximate and came down to an integration of ordinary differential equations. The additional information on these researches can be found in [14].

Recently works on a research of interaction of acoustic waves with a boundary layer by direct numerical methods on the basis of the complete dynamical equations of viscid heat-conducting gas began to appear. In [15] the direct numerical simulation (DNS) was used for a study of an interaction of sound waves with a supersonic boundary layer at Mach number M=4.5. The numerical simulation of a hypersonic boundary layer receptivity to fast and slow acoustic waves at M = 6 is carried out in [16]. In [17] calculations of the boundary layer receptivity to fast and slow acoustic waves with M=4.5 carried out. Calculation and experimental studies of a hypersonic shock layer receptivity to acoustic disturbance at M = 21 have been conducted in [18]. All this studies were carried out only for the interaction of boundary layer with 2D acoustic waves. Apparently, there is a single work [19] which solved the interaction problem of 3D acoustic monochromatic waves with boundary layer. However in it only the case with the fixed sliding angle relative to the front edge of the plate and the just one frequency was considered. Therefore, in this researches on interaction of acoustic waves of different incidence and sliding angles with the supersonic boundary layer at Mach number M=2.0 are conducted.

# 2 Problem statement and basic equations.

The gas flow is described by the known Navier - Stokes, continuity, energy and state equations [20]:

$$\rho^*\frac{\mathrm{d}\mathbf{v}^*}{\mathrm{dt}}=-\,\mathrm{grad}\left(p^*\right)-\frac{2}{3}\mathrm{grad}\left(\mu^*\mathrm{div}\left(\mathbf{v}^*\right)\right)+2\mathrm{Div}(\mu^*\dot{\mathrm{S}}),$$

$$c_p\rho^*\frac{dT^*}{dt}-\frac{dp^*}{dt}=2\mu^*\dot{S}^2-\frac{2}{3}\mu^*\left(\mathrm{div}\left(\mathbf{v}^*\right)\right)^2+$$
$$+\mathrm{div}\left((\mu^*c_p/Pr)\mathrm{grad}T^*\right),$$

$$\frac{d\rho^*}{dt}+\rho^*div(\mathbf{v}^*)=0,\ p^*=\rho^*RT^*$$

Here $\mathbf{v}^*$ –velocity with components $\left(u^*,v^*,w^*\right)$ in $x,y,z$ − directions, $p^*,\rho^*,T^*$ − pressure, density and temperature, $c_P$ − specific heat at constant pressure, $R$ − gas constant, $\dot{S}$ − velocity tensor, $Pr=c_p\mu^*/\lambda^*$ - Prandtl number, $\lambda^*$ − thermal conductivity, $\mu^*$ − dynamic viscosity.

Interaction of a boundary layer with acoustic waves was computed by numerical simulations on the base of the Navier-Stokes equations with using of the ANSYS Fluent software package.

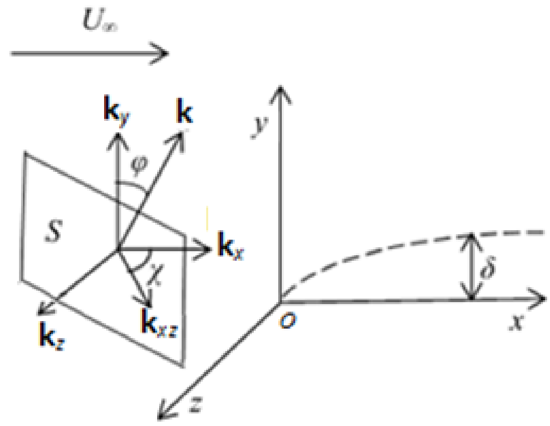


Fig.1 Scheme of an interaction of monochromatic sound waves with the boundary layer

Scheme of an interaction of monochromatic sound waves with the boundary layer of the streamline surface (plate) is shown in Fig.1. The wave front section is represented by the letter *S*. The wave vector **k** of the sound wave has projections: $k_x$, $k_y$, $k_z$ on *x*, *y*, and *z*-coordinates respectively. The wave vector $\mathbf{k}_{xz}$ is the projection of the **k** vector on plate (*xoz*). There are angles χ between vectors $\mathbf{k}_x$ and $\mathbf{k}_{xz}$, φ between $\mathbf{k}_y$ and $\mathbf{k}_{xz}$. Velocity of the running flow is parallel to an axis *x* and is perpendicular the planes (*yoz*). The figure shows a boundary layer and its thickness $\delta$ conditionally. The acoustic wave is periodic with periods: $\lambda_x = 2\pi/\alpha$, $\lambda_y = 2\pi/k$, $\lambda_z = 2\pi/\beta$ on *x, y, z* - coordinates. Thus, the external parameters of the acoustic wave are changed by law $q_i = q_i^0 \cos(k_x x + k_y y + k_z z - \omega t)$, were $q_i^0$ - oscillation amplitude, $\omega$ – angular frequency. Problem is solved in the linear approximation relatively excited disturbances by an acoustic wave.

In Fig.2 the upper part of the computational domain is shown. The lower part is symmetric relatively to the square ABCD. The flow direction is shown in fig.1. The height of the parallelogram AE was selected to avoid of the interaction of shock waves, which forms in the leading edge vicinity of the plate due to viscous-inviscid interaction, with the top side (EHGF). The width AD is taken equal to the wavelength in z-direction $\lambda_z = 2\pi/k_z$ .

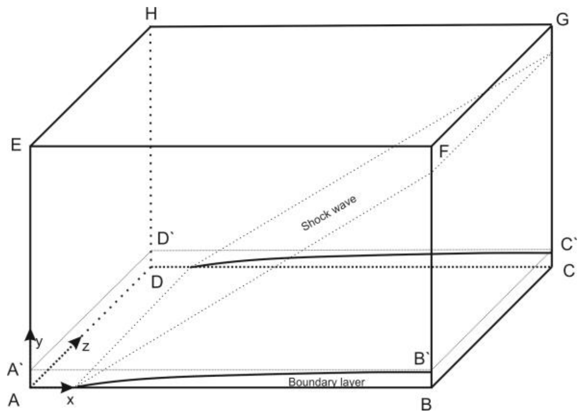


Fig. 2 The upper part of the computational domain

On an entrance (side AEHD) and on the top side flow parameters are set which are composed of the stationary part and parameters of a sound wave:

$$p^* = p^*_\infty + Bp^*_\infty \cos(k_x x + k_y y + k_z z - \omega t)$$

$$T^* = T^*_\infty + BT^*_\infty \frac{(\gamma-1)}{\gamma} \cos(k_x x + k_y y + k_z z - \omega t)$$

$$u^* = u^*_\infty + B\sqrt{\frac{RT^*_\infty}{\gamma}} \cos(\chi)\cos(\varphi) \bullet$$

$$\bullet \cos(k_x x + k_y y + k_z z - \omega t),$$

$$v^* = B\sqrt{\frac{RT^*_\infty}{\gamma}} \sin(\varphi)\cos(k_x x + k_y y + k_z z - \omega t),$$

$$w^* = B\sqrt{\frac{RT^*_\infty}{\gamma}} \sin(\chi)\cos(\varphi)\cos(k_x x + k_y y + k_z z - \omega t),$$

where *B* is a dimensionless pressure amplitude, lengths of wave vectors: $k_x = k\cos(\varphi)\cos(\chi)$, $k_z = k\cos(\varphi)\sin(\chi)$, $k_y = k\sin(\varphi)$,

$k = \omega / [(M\cos(\varphi)\cos(\chi) - 1)\sqrt{\gamma RT^*_\infty}]$,

The Mach number $M = \sqrt{(u^{*2} + v^{*2} + w^{*2})/\gamma RT^*}$ ,

The nonslip boundary conditions (**v**$^*$**=0)** are imposed on the plate surface (PLCB) and the plate temperature corresponds to the adiabatic condition ($\partial T^*/\partial y = 0$).

## 3 Results

The basic calculations were done for the undisturbed Mach number *$M_e$=2,* Prandtl number *Pr=0.73,* parameter $F = 2\pi f \nu_e / u_e^2 = 0.445 \cdot 10^{-4}$ (*f* — frequency in Hertz). The dependence of the dynamic viscosity on temperature was adopted in accordance with the Sutherland's law:

$$\mu^* = \mu_r^* \left(\frac{T^*}{T_r^*}\right)^{3/2} \frac{T_r^* + T_s^*}{T^* + T_s^*},\ T_s^* = 110^\circ K,\ T_r^* = 164^\circ K.$$

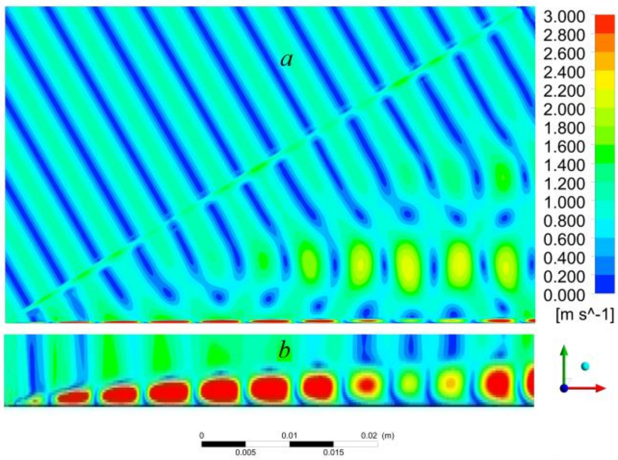


Fig.3 Instantaneous full velocity perturbation contours induced by planar free-stream acoustic wave (χ=0°, φ=30°)

In Fig.3a the instantaneous contour of the longitudinal velocity perturbation induced by planar free-stream acoustic wave is shown in the area between the surface plate and the top side of computational domain for χ=0°, φ=30°. This contour is shown more detail near the plate in Fig.3b. It can be seen that at the some distance from the plate edge a periodic structure under shock wave coincides with the structure above the shock wave practically. It tells about a weak intensity of the jump created by a leading edge area of the thin plate.

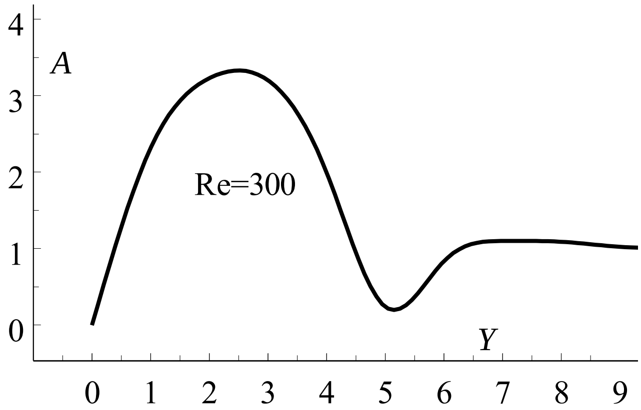


Fig.4 Distribution of the fluctuations amplitude of the longitudinal velocity nearby of a plate, χ= φ =0°

In Fig.4 the distribution of the fluctuations amplitude of the longitudinal velocity near by the plate (in the boundary layer) for χ=φ=0° is shown, were $Y = y^* \sqrt{u_e / \nu_e x^*}$ , $\mathrm{Re} = \sqrt{u_e x^* / \nu_e} = \sqrt{\mathrm{Re}_x}$ . There are similar distributions for other wave's parameters. It is important to note that the perturbation amplitude within the boundary layer is greater than the amplitude of the external acoustic wave in several times, that is consistent with conclusion, which is based on the stability equations of parallel flows [14]. The maximum value of the amplitude, $A_{max}$, is located near $Y_{umax} \approx 2.5$.

In subsequent figures the results about the influence of orientation angles of acoustic waves on maximal values of oscillation amplitudes, $A_{max}$, of the full velocity,

$(\tilde{u}'^* = (u'^* \cos(\chi) + w'^* \sin(\chi))\cos(\varphi) + v'^* \sin(\varphi))$ ,

divided by its value in the initial acoustic wave, $A_0$, will be shown.

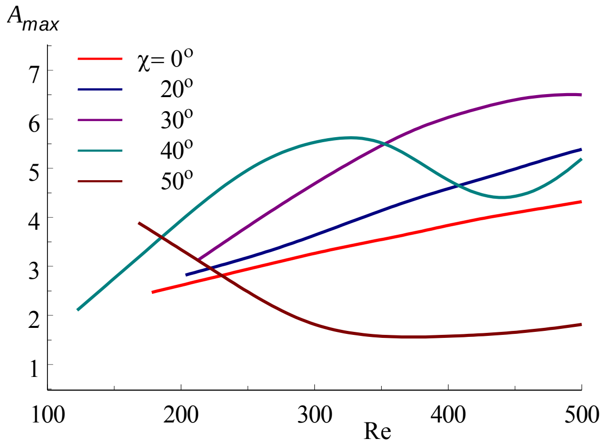


Fig.5 The maximum amplitude of velocity oscillations in depending on the Reynolds number at different sliding angles, φ=0

The dependence of the maximum disturbance amplitudes of a full velocity, on the Reynolds number at the different sliding angles is shown in Fig.5. At the small sliding angles the perturbations amplitudes increase monotonously at least up to Re=500. With an increase of a sliding angle there is a maximum (at χ >30°) in this dependences. It is displaced to a leading edge of a plate with increasing of χ. In this case the amplitude increase in the field of Reynolds numbers Re >450 is connected with the boundary layer instability.

The dependence of maximum amplitude on the sliding angle at fixed positions *x* is shown in Fig.6 for three Reynolds numbers. From these data it is visible that at the fixed Reynolds's number and frequency there is a critical value of an sliding angle $\chi^*$ at which the disturbances excited by a sound wave are maximal. With increasing of the Reynold's number the critical sliding angle decreased.

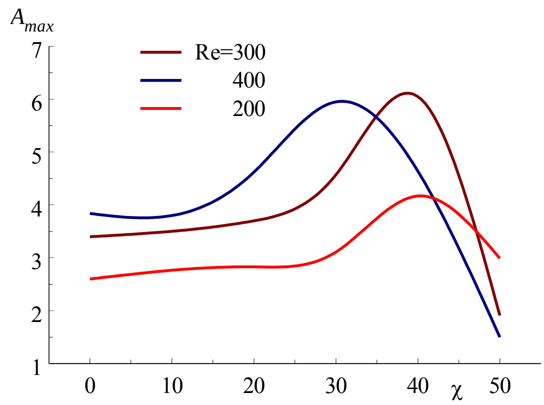


Fig.6 Amplitudes maximum of the full velocity in depending on the orientation angle, φ=0

Influence of the incidence angle of the sound wave on the velocity oscillations intensity in the boundary layer is shown in the next figures.

The dependences of the maximum amplitudes of velocity oscillations on the Reynold's number at different incident angles for χ=0 are shown in Fig.7. Here it is important to note that in the dependence of the maximal amplitude at φ=45 there is no its increase in the field of Re>450 unlike similar dependences at χ=40 and 50 at φ=0 (Fig.5). However, in general, the dependences of the maximum amplitudes of velocity oscillations from the Reynolds number at different incidence angles for χ=0 similar the dependences of the maximum amplitudes of velocity oscillations from the Reynold's number at different sliding angles for φ

=0 (Fig.5). With the increase of the incidence angle there is a maximum (at φ >15°) in this dependences. It is displaced to a leading edge of a plate with increasing of φ.

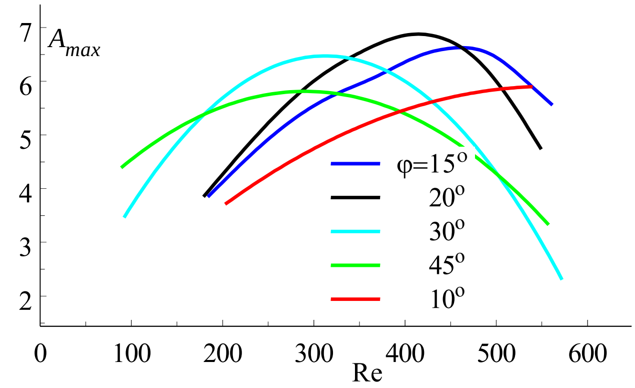


Fig.7 The maximum amplitude of velocity oscillations in depending on the Reynold's number at different incidence angles, χ=0

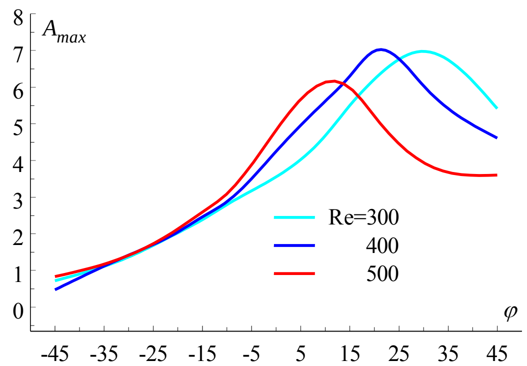


Fig. 8 Amplitudes maximum of the full velocity in depending on the incidence angle φ, χ =0

The dependence of maximum amplitude on the incidence angle at fixed positions Re is shown in Fig. 8 for three Reynolds numbers. From these data it is visible that at the fixed Reynolds number and frequency there is a critical value of an incidence angle $\varphi^*$ at which the excited by a sound wave disturbances are maximum, similar to dependences of Fig.6. As in results of Fig.6 with increasing of the Reynolds number the critical incidence angle decreased. Unlike dependences of the Fig.6 dependences of the Fig.8 are presented not only in field of the positive angle values but in field of their negative values. This is due to the physical peculiarity of the sliding and incidence angles. It is clear that $A(\chi)=A(-\chi)$. Negative incidence angles indicate that the plate is irradiated by an acoustic wave from below and the results shown in the Fig.8 correspond to the area above the plate, or on the contrary. Data of Fig.8 show that the oscillations exaltation in the boundary layer by the sound wave more efficiently if the plate is irradiated from above.

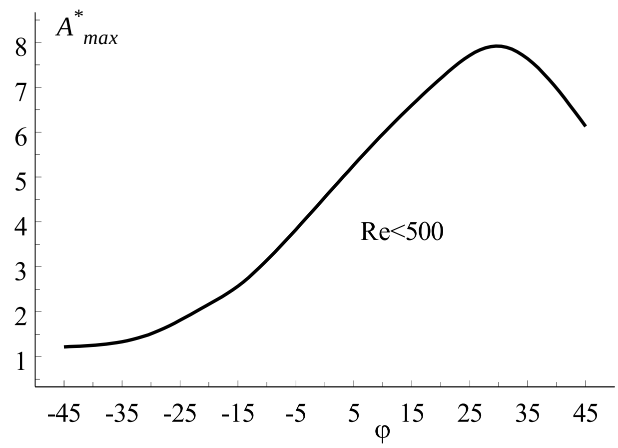


Fig.9 The dependence of the maximum value of the fluctuations amplitude in the area 100<Re<550 on the incident angle

From data of fig.7 it is visible that in the area Re< 550 and at the fixed incident angle the disturbances amplitude $A_{max} \leq A^*_{max}$. The dependence of the maximum value of the fluctuations amplitude within of the interval of Reynolds numbers 100<Re<550 $A^*_{max}$ on the incident angle is shown in Fig.9. Comparison of these data with the results of Fig.7 shows that the most intensive oscillations are excited by the sound wave with the incident angle $\varphi \approx 25^o$ at Re≈ 350.

## 4 Conclusion

In the paper on the basis of the direct numerical simulation the supersonic flow around of the infinitely thin plate which was perturbed by the acoustic wave was investigated. Calculations carried out in the case of small perturbations at the Mach number $M_e=2$ and Reynold's numbers Re<600. Two types of acoustic waves were investigated: sliding (φ=0), two dimensional incident waves (χ=0). As a result of calculations it was received:

1. The velocity perturbation amplitude within the boundary layer is greater than the amplitude of the external acoustic wave in several times.
2. At the small sliding and incidence angles the velocity perturbations amplitude increased monotonously with Reynold's numbers.
3. At rather great values of this angles there are maxima in dependences of the velocity perturbations amplitude on the Reynold's number.
4. At the fixed Reynolds's number and frequency there are a critical values of the sliding and incidence angles at which the disturbances excited by a sound wave are maximum.

5. The oscillations exaltation in the boundary layer by the sound wave more efficiently if the plate is irradiated from above.

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