Risk assessment in estimating the capitalization rate

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Abstract: - In estimating the market value of a property by income approach, the determination of the capitalization rate is ordinarily conducted through analogical process. The procedure is based on the identification of price-earnings ratio of similar investments. The analogy refers to the risk and the duration of the investment. However, in all cases there still remains a rate of uncertainty that significantly affects the final estimation of the property. This paper proposes a methodology which removes any uncertainty when evaluating the cap-rate. The aim is achieved through the combination of the formal logic of the Ellwood's model and the Real Options Analysis. The algorithm developed has been applied to a sample relative to 57 Italian cities, considering annual prices from 1967 to 2012. The results highlight the validity of the model and the easiness of use. The work must be attributed equally to the three authors.

Key-Words: - Ellwood, Real Options, cap-rate, risk, real estate, investment.

1 Introduction and objectives

When the estimation of the market value of property is based on the capitalization of income, the choice of capitalization rate can be made through either a statistical or analogical procedure. The latter, which is the most widely adopted, is usually a comparison between an assumed return rate on investments similar to those of the property to be evaluated. The analogy specifically concerns two aspects: the risk and the duration of the investment. The assumed rate must be correctly defined, either up or down, in order to take into account the differences between investment risk and the different role played by the return rate in relation to the function of the capitalization rate.

In current literature, there is an ample discussion on the procedures that are intended to streamline the estimation by analogy of the capitalization rate. Interesting reflections on the applications of the Ellwood model on the estimation of capitalization rates can be found in [1]. Jud G.D. and Winkler D.T. [11] use the weighted average cost of capital (WACC) and the capital asset pricing model (CAPM) to develop a model for estimating capitalization rates. Del Giudice V. [7] makes a classification of the estimation procedures of the rate. Epley D.R. [10] deals with the Band of Investment.

In all cases, however, there still remains a rate of uncertainty that significantly affects the final estimation of the property. This is due to the inverse relationship that ties the rate to the result of the estimation, so that – as it is well-known – a slight change, increasing or decreasing, of the capitalization rate corresponds to significant inverse variations in the estimations.

This work aims to reduce the uncertainty inherent in estimating the capitalization rate by converting it into risk. This is achieved by combining the logic at the bases of the Ellwood model and Real Options Analysis (ROA), with the main references being discussed in the following paragraphs [12, 14]. In detail, the property investment pattern of the Ellwood model, obtained from the sequence of the financial income and expenses related to the established availability period of the property, combined with the risk analysis of the ROA, makes it possible to anticipate the evolution of the investment value up to the year in which it is convenient to recover the initial capital. The work considers a model already published [12]. In this paper it is applied to a database related to the period 1967-2012. In this way, the application is immediately available, taking into account the latest developments in the Italian housing market.

The choices derived from the analysis carried out and the verification of the reliability of the model that is proposed are made via the implementation of the model to the estimation of the market value of the properties that make up a sample of the study as well as the subsequent measurement error. The result is a capitalization rate that has value on a broad area identified – for example – with the centre or the periphery of a city. Accordingly, this rate should be corrected when having to take into account both the effects of specific factors of the area as well as the intrinsic characteristics of the property subject of the estimation. In Medici [13], in order to land assets, that the rate is less where the convenience offered by the property is greater. The proximity to major centers and lines of communication, the health and beauty of the area, the location, the peaceful nature of the population as well as elevated social harmony all have a significant influence. The rate is also reduced by the size of the property, which makes the purchase accessible to more people, the presence of a large and prosperous class of small farmers or owners of small tenants or wealthy sharecroppers. However, the rate, tends to increase as a result of causes opposite to those described. High rates, e.g. of 7%, and 8%, are found in malarial areas, low population, without any lines of communication. The rate also tends to increase, and therefore the land value decrease, where a restless rural proletariat is strong and the trade-union of workers and farmers, small tenants is active. Not simply in terms of reduced competition among workers, but also due to the creation of new social conditions in rural areas, which tend to reduce these indirect benefits of which the authors of rural estimation discuss.

2 Discussion on the Ellwood model and Real Options Analysis

In the Ellwood model, the capitalization rate is sought from the equation that adds up the assets and the liabilities of a typical property investment [9].

The revenues are from the net income received by an ordinary operator during the course of the availability period of the property (projection period) as well as the residual value investment, consisting of the purchase price which should be added or subtracted from the revaluation or devaluation percentage at the end of the availability period of the property. The residual value is clearly recovered when having the disposal of the investment.

The costs, in turn, are given by the expenses required to start the investment. These include the purchase price of the property, the brokerage fees as well as the expenses associated to ownership transfer (tax and notary fees). The operating costs are already counted in determining the net, annual average, ordinary and continuous incomes, corresponding to the net fees earned after the expenses of the main part.

In addition, since in most cases, whoever is purchasing a property either does not have the entire sum or may have it but decides not to invest it and prefers external financing (bank loan/mortgage), the amount for the purchase – which in the latter case differs from the purchase price – is calculated as the sum of equity capital (self-financing) and the share of debt capital, which is part of the expenses on the balance sheet in the form of the depreciation rate of the loan/mortgage.

In the Ellwood model, the revenues and expenditures, due to being attributable at different times, become financially homogeneous through the reduction of the amounts of individual items to year zero by using the internal return rate of the property investment.

From the financial analysis, the capitalization rate is then derived as the relationship between the ordinary producible income from the property and the estimated value of the property.

Real Options Analysis is a technique for estimating investments that can be successfully used to manage the uncertainty related to possible changes of scenario.

Compared to the static approach, which considers the updating of cash flow of the future investment for the most likely scenario, ROA, where the uncertainty can be transformed into risk, makes it possible to carry out a risk analysis of the various project solutions, known as options.

There are multiple analytical formulations of the ROA. In this work, the binomial [4] paradigm is used, which develops the changes of the initial values of the investment through multiplicative probability, defined by the coefficients u>1 and d<1, which represent, respectively, the evolution of the initial state towards either a favourable or unfavourable scenario.

The size of the coefficients u and d is the result of the investment risk analysis, statistically calculated by a dispersion index that, in the case of a normal distribution of the variable under analysis, coincides with its standard deviation (σ).

Having estimated the risk, the definition of the possible evolution states occurs with the mathematical expressions:

$$\begin{cases} u = e^{\sigma \cdot \sqrt{dt}} \\ d = e^{-\sigma \cdot \sqrt{dt}} \end{cases} \tag{1}$$

where:

e = Napier number;
σ = standard deviation, also called – in ROA – volatility;
dt = considered time interval between the successive scenario evolutions.

With the use of the coefficients u and d, the initial investment has a typical tree-like structure (scenario tree), derived from the analysis grid [5], [15].

A decision tree is built after the risk analysis. This is the stage where feasible options are identified and the corresponding values quantified. The decision tree makes it possible to measure the impact, on the financial analysis, of the possible development options of the investment. The impact measurement requires the preliminary definition of a maximization function, which conforms to the particular type of option identified. With this function, in the year when the option is exercised and for every possible scenario in that year, the comparison of the present value of the investment "with" and "without" the option is carried out.

The operation described is performed for all the n scenarios of the year of feasibility of the option, making it possible to establish a vector of the majors, of dimension nx1, which is then discounted to the current scenarios following the scenario tree from right to left. The calculation is performed by weighting the elements of the vector with the coefficients of risk-neutral probabilities (p and $1-p$), relating to the current situation, the outcome of the weights using a discount rate (r_i). The result is the present extended value of the investment, a value that includes the effect of the option or options that may be exercised.

3 The basic assumptions

One hypothesis is that the property is purchased with the entire sum. This eliminates three variables from the termination equation of the financial analysis of the Ellwood model: the partially financed capital, the interest rate for the recovery of the loan, as well as the duration of the depreciation period. The eligibility of the simplification is due to the low influence that the three variables have on the final result, especially when not determining the performance of a property investment, but rather a capitalization rate.

Another hypothesis assumes that the availability period of the property, due to it deriving from the construction and solution of a maximisation function of the profits generated by the investment, is an endogenous variable of the model. In fact, the maximization of profits is a primary objective of any economic entity that decides to invest capital, and therefore also of an ordinary investor. The maximisation function used in the model implements the risk analysis of the investment.

Another possibility includes the invariability of income. In fact, as confirmed through the application of the model to several case studies, the duration of the investment is roughly equal to the normal length of a lease. In Italy, current "ordinary" contracts (therefore, excluding subsidized contracts), set out, for buildings for residential use and those for commercial use, respectively, the formulas of 4+4 (eight years) and 6+6 (twelve years). During these periods, the change in rent is not influenced by market trends, and there is only an annual adjustment equal to 75% of the variation of the annual consumer price index. A study on how the effects of macroeconomic factors have been generated on the economic cycles of the housing market of four California cities [8] showed that leases complicate the transmission mechanism of any "shocks" to the external application (such as an unexpected growth in employment) on the values of the lease. The contracts determine the rental market, a significant "delayed" impact compared to the effect felt on real estate transactions.

These contracts do not include the variability of the agreed income, thus, it is to be expected that, at the end of the availability period of the property, the actual rent (i.e. based on the invariability of prices) remains constant.

The implemented risk analysis objectifies the estimation of the capitalization rate, eliminating the remaining uncertainties in the definition of the capital property appreciation rate as well as the discount rate in the financial analysis. The assumption is that the riskiness of the property investment can be explained by the volatility of the time series of the annual rates of appreciation or depreciation of property values. The time series of the rates can be constructed from the annual average property values for the city and homogeneous areas, provided by databases for sufficiently long periods, which are therefore statistically significant. The use of verification tests has highlighted that the distribution of the time series of the annual rates agrees with good approximation to the assumption of normal development, required for the development following the analyses.

The volatility of the series of the appreciation or depreciation rates of property values, or the investment risk, may therefore be represented in an effective measurement of its standard deviation (σ), calculated as a percentage of the average in the series.

4 Description of the model

In its formal and logical articulation, the new model is defined by five equations, presented in Table 1.
On the basis of the aforementioned assumptions, the Ellwood financial statement for a property investment can be written using eq. (1) in which appear, the first term, the revenues from the investment during the availability period of the property (m), obtained from the sum of the deferred and constant annual incomes (R) and increased by the property value at the end of the availability period (*projection period*), consisting of the purchase price (P) re-evaluated with the coefficient r_v. All the items of the first term are discounted at a rate (r_i). The second term is given by the initial investment cost (K), consisting of the purchase price of the property, the brokerage fees and associated costs of ownership transfer.

$R \cdot \left[\frac{(1+r_i)^m - 1}{r_i \cdot (1+r_i)^m} \right] + \frac{P \cdot (1+r_v)^m}{(1+r_i)^m} = K$	(2)
$VA = R \cdot \left[\frac{(1+r_i)^m - 1}{r_i \cdot (1+r_i)^m} \right] + \frac{P \cdot (1+r_v)^m}{(1+r_i)^m}$	(3)
$VA' = R \cdot \left[\frac{(1+r_i)^m - 1}{r_i \cdot (1+r_i)^m} \right] + \frac{P \cdot [(1+r_v)^m - 1]}{(1+r_i)^m}$	(4)
$\left[P \cdot r_c \cdot (1 - c_{vacancy} - c_{income}) - P \cdot 2/3 \cdot (s_{reint} + s_{mai}) - P \cdot c_{property} \right] \cdot \left[\frac{(1+r_i)^m - 1}{r_i \cdot (1+r_i)^m} \right] + \frac{P \cdot (1+r_v)^m}{(1+r_i)^m} = P \cdot (1 + s_{broker} + s_{transfer} + s_{notary})$	(5)
$r_c = \frac{\left[(1 + s_{broker} + s_{transfer} + s_{notary}) - \frac{(1+r_v)^m}{(1+r_i)^m} \right] \cdot \left[\frac{r_i \cdot (1+r_i)^m}{(1+r_i)^m - 1} \right] + \left[2/3 \cdot (s_{reint} + s_{mai}) + c_{property} \right]}{(1 - c_{vacancy} - c_{income})}$	(6)

Table 1 – Equations that describe the model

In the *binomial* system adopted for the analysis of the real options, once the standard deviation (σ) and the parameters u and d have been estimated, the stochastic evolution of the present value of the investment (VA) is defined (VA is equal to the VAN of the property investment with the exclusion of the initial costs K). As reported in Table 1, this evolution coincides with the first member of eq. (2) and can be written using eq. (3).

It should be noted that the purchase price (P) of the property – also present in eq. (3) – is a constant whose exclusion from the calculation does not produce errors in the probabilistic evolution of the investment which can be set out in the *scenario tree*, but it simplifies some mathematical manipulations. Therefore, in eq. (4), the value of the current year estimate (VA') is set equal to the sum of real financial cash flow (gross income generated from the rental of the property) and virtual flows (annual revaluation or depreciation of the capital). The latter being flows which are so called because they are not collected every year, but gradually accumulate in the residual value that is received in year m of mobilization of the investment.

For the construction of the *scenario tree* based on the binomial approach, we assume that all the parameters of eq. (4) are known, with the exception of the availability period of the property (m). On this condition, it is possible to show that, given the option for which the sale will be exercised in the year in which the option itself will be more cost effective, there is a unique relationship between the volatility value of the investment (σ) and the availability period of the property (m), to offset the uncertainty related to the identification of the *projection period*. The demonstration has calculated the present value of the cash flows in the first year, then the second, then third and so on, assuming that the option of selling the property, is carried out in a worst situation, i.e. along the branch of the worst evolution of the property investment. The calculation shows that, for a given volatility value, the d^{i} *VA'* value of the investment – obtained from the most pessimistic scenario, by gradually increasing *m* – has a trend that uniquely defines a maximum value.

Fig. 1 shows the change in the d^{i} *VA'* value for the different volatility values, assigning the parameters of equation 4 with reference to a case study. Using a simple algorithm that expresses the maximization function of the profits achieved from that investment, there is the maximum of the function, or the maximum d^{i} *VA'* and the corresponding year *m* of the disposal of the property.

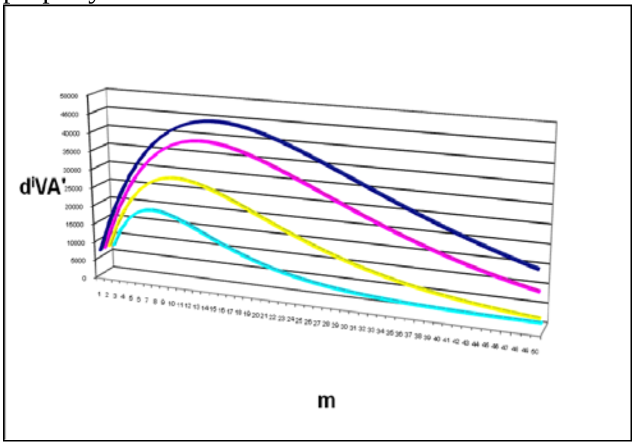

Fig. 1 – Development of the d ^{i} VA' value varying volatility (σ)

Built with the same data as Fig. 1, Fig. 2 shows that the relationship between σ and m is consistent with the expectations so that, for an increase of σ, and hence the riskiness of the investment, there is a reduction of the availability period of the property. Chu Y. And Sing T. F. [2] have highlighted that in the property market, investors are contrary to exercising the deferment in light of possible future developments, when there are concrete and actual conditions that threaten the expected revenues in the short-term.

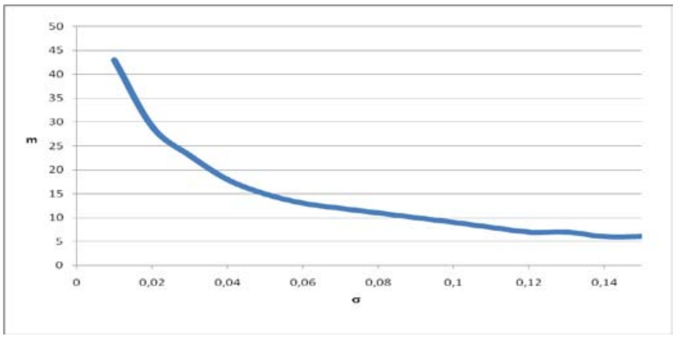

Fig. 2 – Relationship between the availability period (m) and volatility (σ)

Analysing the evolutionary path described by the branch of the scenario tree immediately above the most pessimistic one (ud^{i-1} *VA'*), it can be noted that the year in which the exercising of the option to allow for the sale with maximum profits is not different. Similarly, when analysing the branches of the tree, the paths that describe progressive evolutions of more favourable scenarios ($u^2d^{i-2}VA'$, $u^3d^{i-3}VA'$, $u^4d^{i-4}VA'$,..), there is always the same year *m* that is more convenient to dispose of the investment. Year *m* is therefore the time when, regardless of the more or less favourable scenario, the property should be sold. Due to the results described and having fixed the cost-effectiveness as the sole criterion for defining the year of disposal of the investment, the sale cannot be treated as a real option. In fact, an option may be defined as the *ability* of the decision maker to exercise a choice under certain evolutionary conditions, choice upon which the analysis of real options is able to assign a value. In the analysis carried out, however, the sale in year *m* does not have a more convenient alternative scenario or branch of the tree. Therefore, it is a *requirement*, for the normal investor, to sell in that year. This creates the return to eq. (2) after determining, as described, the year m.

In eq. (2), it is possible to make the expenses of the main part explicit, in order to obtain, from the rent, the net income and the initial investment costs (*K*).

r_c denotes the gross capitalization rate, defined as the relationship between the rent (*Ca*) and the purchase price of the property (*P*), the balance sheet of eq. (2) can be rewritten with eq. (5), with the taxes and expenses being listed in Table 2.

$c_{vacancy}$	percentage of losses due to vacancy and write-offs (% of R)
c_{income}	percentage of taxes on income (% of R)
s_{reint}	percentage of the annual fee for annual reintegration for the renewal of the property capital at the end of its useful life (% of construction cost)
s_{mai}	percentage corresponding to the costs of extraordinary maintenance, administration and insurance (% of construction cost)
$c_{property}$	percentage corresponding to property tax (% of P)
s_{broker}	percentage corresponding to the costs of marketing the property, brokerage fees (% of P)
$s_{transfer}$	percentage corresponding to the property transfer tax (% of P)
s_{notary}	percentage corresponding to the notary fees (% of P)

Table 2 - taxes and expenses that appear in Equations 5 and 6 of Table 1

The coefficient for calculating the reintegration rate (s_{reint}) is obtained, as known, with the formula of the final accumulation of the annual constants, delayed and limited. Once the data on the remaining life of the property (v_r) has been obtained, the coefficient is determined according to the discount rate (r_i) using the following equation:

$$s_{reint} = \frac{r_i}{(1+r_i)^{v_r} - 1} \qquad (7)$$

In eq. (5), the expenses relating to the shares of the reintegration and extraordinary maintenance, administration and insurance are calculated as a percentage of the cost of constructing the property, estimated as equal to 2/3 of its market value. The determination of these items does not create any problems due to it normally being revealed by the data collection and measurement or calculation of the magnitudes involved.

Finally, eq. (5) is rewritten in the form of the eq. (6) where the capitalization rate is explained as the unknown of the model.

Particular attention is required in estimating the rates r_v and r_i. The first, which is the annual rate of appreciation or depreciation, in this work is determined by a procedure that uses the time series of property values derived from easily accessible sources. The rate is then defined as equal to the weighted mobile average of the series calculated over a period corresponding to the duration of the property economic cycle, an average of five years. The macro-economic factors which depend on the real estate cycles are designed by R.H. Eldestein and D. Tsang [8]. The definition of the rate r_v is objective, differentiated by city and homogeneous area, and reflects the evolutionary trend of the phenomenon. The weighted mobile average, through the use of a linear growth factor, makes it possible to assign greater weight to the information which is more recent and next to the estimation.

The discount rate (r_i) coincides, from a financial perspective, with the internal return rate of the investment. However, in the Ellwood model as well as this new model, it is not obtained as a result of financial analysis, but is introduced by the operator as a well-known fact among the input variables. The unknowns in the equation are consequently reduced to only one, the capitalization rate (r_c). The estimation of r_i is actually carried out by the criterion of *opportunity cost*, assuming as a discount rate, the return rate on an alternative investment to that of the analysis, similar in terms of risk and duration.

In property investments, it is common practice to choose government bonds, as alternative use of the capital. However, the analogy between the property market and public bonds is, in fact, an approximation that involves accurate corrections to the return rate of government bonds in order to obtain the discount rate for the property investment. In conditions of economic stability, it could be argued that the ordinary economic subject who invests money in property, in relation to the similar investment in government bonds which are normally considered risk-free, has a greater appetite for risk as well as managerial capacity to manage the property. It is therefore clear that continuing with this logic would have led to an estimation model that was no different from the others, applied to the indirect estimation of the capitalization rate.

An alternative type of investment, which is similar in terms of risk and duration of the property investment, is that of closed real estate funds. The market, thus, is a share one where there is a variety of financial products associated with a variability of returns, leading to the exclusion of adopting property funds as an alternative investment.

This leads to the idea of using a discount rate related to parameters that are already defined and known by the model, which proposes, through the following conditions:

- the choice of the alternative investment cannot be separated from the expected return for the property being valued, for which the rate of appreciation or depreciation (r_v) is already defined;

- an alternative investment should be treated in relation to the risk that characterizes the expected return and that during the implementation of the model, it is analyzed and expressed through the calculation of the volatility (σ) of the investment. An increase in the cost of money and the market presence of other forms of investment with more attractive yields could adversely affect the demand for properties. The capitalization rates could increase greatly if investors require a higher return from the property market in order to compensate for the rising interest rates [3].

For the two conditions mentioned, the calculation of the discount rate r_i can be carried out through the choice of a linear combination of the two parameters σ and r_v, which can represent an extreme limit of the actual investment return and risk. Considering that the standard deviation (σ), with respect to a normal type distribution, makes it possible to know the range – centred on the average – around which a number of observations falls, and given that the range of ± 3 σ covered 99.73% of the observed measures, it follows that the discount rate sought can be obtained with the eq. (8):

$$r_i = r_v \cdot (1 + 3 \cdot \sigma) \tag{8}$$

5 Application of the model

The model described – as previously stated – leads to the definition of an average capitalization rate that has a value over a wide area, identified with either the centre or periphery of a city. It follows that the capitalization rate obtained with the new model will be correct when having to take into account – if there are any – the effects, on the final value, of specific local factors as well as intrinsic characteristics of the property to be estimated.

In order to test the reliability of the new model, it was applied to the estimation of a typical property located in either the centre or periphery of fifty-seven Italian cities, considering annual prices from 1967 to 2012. The values obtained were compared with the corresponding data on estimated market values of the Property Market Observatory (OMI) of the Inland Revenue, with the differences being measured.

For the economic and physical characterization of the property, it should be stated that:

a) the tax and expenditure items in Table 2 were assigned the mean rates and fixed rates;

b) an observed price was assumed the average value of the data collected by the OMI in the reference area (centre or periphery), for a typical property for residential use and under normal conditions;

c) the gross rent was well-defined in the OMI data. In order to verify the model, it is not necessary that OMI rates fully comply with the actual market data. This has often been criticised, but it is sufficient that there is a relationship between price and fee (reported by the OMI), that is representative of the market capitalization rate.

The risk of the investment has been defined by determining the standard deviation (σ) as a percentage of the average of the time series of the rate r_v , a series which was built for both the centre and suburbs of each of the fifty-seven cities considered, from a sample of property values relating to residential use. The construction of the series has been created by "purifying" the data of the changes due to inflation.

The rate r_v, used in the statements, was calculated as the weighted mobile average of the same time series of the rate r_v.

The parameters defined were used in estimating the discount rate (r_i) as well as in defining the availability of the property (m).

The capitalization rate r_c, was subsequently determined to the estimation of the value of the typical property and the calculation of the residual value between the last value and the price of a similar typical property type as observed in the OMI data.

Fig. 3 shows the measured residues on the typical properties identified in the two areas, the centre and suburbs of the city, grouped for the three geographical areas: Northern Italy, Central Italy, the South and the Islands. The average total residue is 15.60%, with it being slightly lower in the "centre" of the city (14.16%) and higher for the "periphery" (17.05%). These values highlight a good reliability of the new model.

The histograms in Fig. 4 and 5, indicate the variability of the capitalization rates (r_c) and the variability of the availability period (m) of the *typical* property determined on the pooled data of the 57 cities.

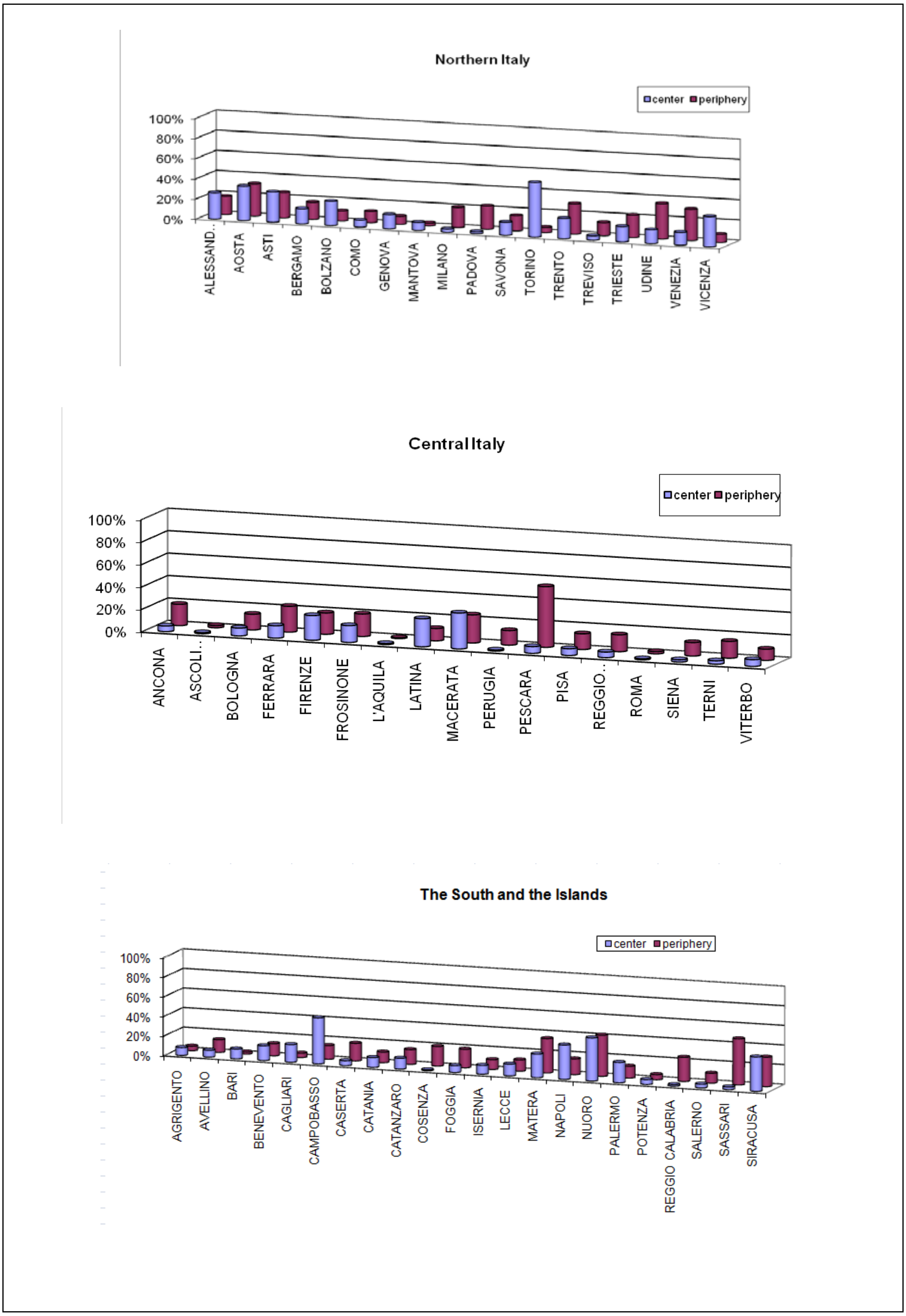

Fig. 3 - Measured percentage of the residuals between the estimated value and the OMI value of the typical property in the cities studied

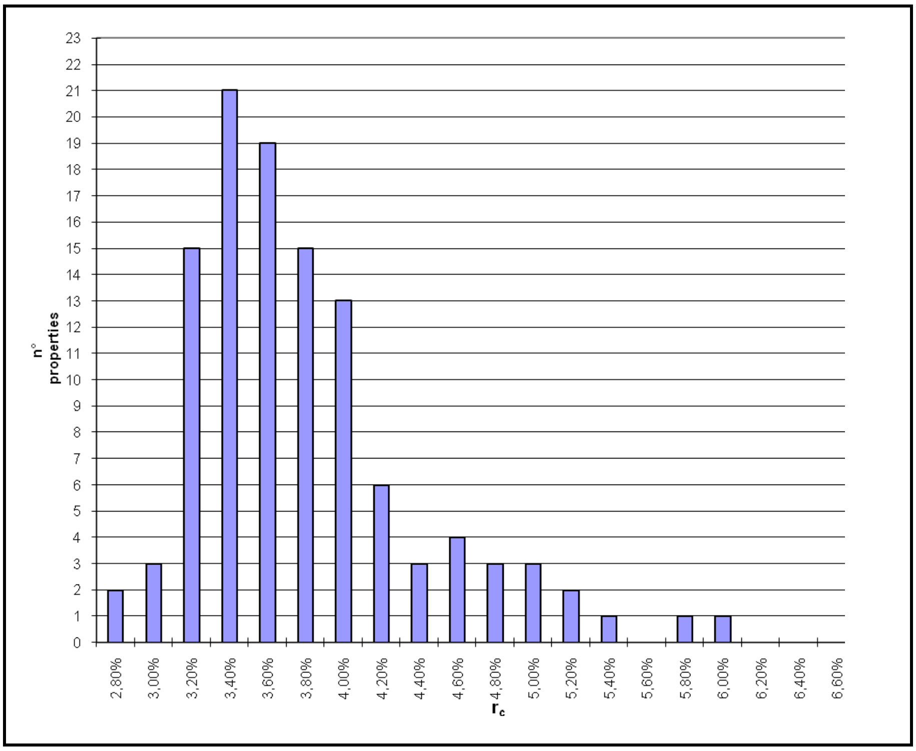

Fig. 4 – Capitalization rates in the cities studied

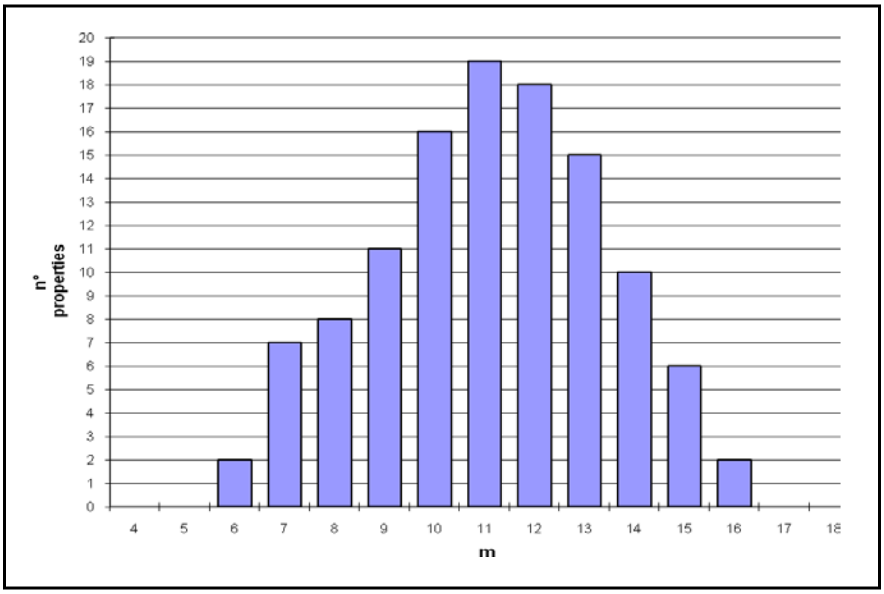

Fig. 5 – Availability period (m) of the typical properties in the cities studied

6 Conclusions

The indirect estimation of the market value of a property, based on the capitalization of income, has two fundamental assumptions: 1) the equivalence of the market value with the result of the relationship between the ordinary income retractable from the asset and the capitalization rate, 2) the presence of the conditions that allow for the prediction of both the income as well as the rate. It is also essential that the data and information is available in order to determine the rates and incomes.

In the event that there is the possibility to use the rents of properties similar to the one estimated, and thus determine the income to be capitalized, the model outlined in this paper makes the choice of the rate an objective one. When the direct estimation of the rate is not feasible, but it is possible to build a time series of the variations in rates of property values, the model provides a capitalization rate of general validity on the reference area, in order to adjust it when there is the need to take into account the effects of specific factors as well as the intrinsic characteristics of the property estimated.

The objectification of the choice of the rate is due to the transformation, into risk, of the residual uncertainty in some exogenous variables to the balance sheet of the property investment. The operation is carried out through the scenario analysis of Real Options, with the risk being related to the forecast of the future revaluation or devaluation of the property capital. The risk analysis has also allowed for the unique determination of the duration of the investment, corresponding to the availability period of the property. It is up to the estimator whether to use the parameters to estimate the start-up expenses (notary fees and property transfer tax) and management of the property investment.

In the risk analysis, the use of a discrete binomial approach to describe the evolution of the investment over time, simplifies the process and relieves the formalizations that mark a continuous evolutionary model. The latter, as it is known, return the infinite states that the variable can take in each scenario and inside the variation cone, eliminating the gaps present in a discrete model. However, in this work, since the choice of divestiture is generated exclusively by the observation of each of the worst states in each scenario, it was considered not to unnecessarily complicate the model with a continuous approach.

The consistency - with real market values – of the results obtained from the experiments, proves the reliability of the model in practical estimation applications.

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