A Method for the Evaluation and Selection of an Appropriate Fuzzy Implication, by using Statistical Data

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Abstract: - In classic logic, there exist an implication of the form $p \Rightarrow q \equiv n(p) \vee q$ (where n(p) is the negation of p and $\vee$ the maximum). If we consider the fact that the propositions p and q take only the values 0 and 1, then the values of the classic implication are well-defined. In fuzzy logic, where the proposition can take any value in the closed interval [0, 1], there are infinite number of fuzzy implications which can be used; hence, a method of selecting the most appropriate implication is required. In this paper, we propose a method of evaluating the different fuzzy implications using available statistical data. The choice of the appropriate implication is based on the deviation of the truth value of the fuzzy implication from the real values, as described by the statistical data.

Key-Words: -Fuzzy implication, statistical data, air passenger demand, GDP.

1 Introduction

We know that the implication in classic logic depends only on whether the premise is true or false. That is, whether the syllogism (reasoning) is true or false depends solely on if the premise and the conclusion is true or false.

Every proposition in classic logic has two values 0 or 1, which is *true* or *false*, *holds* or *does not hold.* Let us suppose we have two such propositions p and q. We symbolize the conjunction (AND) of the propositions with $\wedge$ and the disjunction (OR) with $\vee$, while $\neg p$ is used to symbolize the negation of p (i.e. NOT-p).

The conjunction $p \wedge q$ is true, if and only if both propositions p and q are true. In such a case, it holds that $p \wedge q = \min\{p,q\}$, (Table 1). Indeed, let p be the proposition *"The number 2 is prime"* (true) and q the proposition *"The number 6 is a multiple of 2"* (true). Then, the conjunction $p \wedge q$: *"The number 2 is a prime (true) and the number 6 is a multiple of 2 (true)"* has truth value equal to 1.

The disjunction $p \vee q$ is true, if one of the two propositions is true, that is if it holds that $p \vee q = \max\{p,q\}$, (Table 1). Indeed, let p be the proposition *"The number 3 is an integer"* (true) and q the proposition *"The number 16 is a multiple of 5"* (false). Then, the disjunction $p \vee q$ *"The number 3 is an integer* (true) *and the number 16 is a multiple of 5* (false)*"* has truth value equal to 1.

Let us consider the proposition p *"The population of Portugal is less than that of China"* which is true. The negation of proposition p, i.e. the proposition $\neg p$ *"The population of Portugal is greater than or equal to that of China"* is false.

For determining the truth value of an *implication*, (denoted as $\Rightarrow$) between two propositions p and q (we assume the implication $p \Rightarrow q$ i.e. the proposition p implies the proposition q), it would be enough to determine the truth value of the conjunction $\neg p \vee q$ (Table 2).

Table 1: The conjunction ($\wedge$) and the disjunction ($\vee$) of the propositions p and q [1]

p	q	$p \wedge q$	$p \vee q$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

Table 2: The implication ($\Rightarrow$) of the propositions p and q [1]

p	q	$\neg p$	$\neg p \vee q$		$p \Rightarrow q$
0	0	1	1		1
0	1	1	1	$\rightarrow$	1
1	0	0	0		0
1	1	0	1		1

From the last column of Table 2, it can be seen that the implication $p \Rightarrow q$ is always true, except in the case where the proposition p is true and the proposition q is false, i.e. the case where from a false premise, we arrive at a false conclusion. From Table 2, we also see that whenever we start from a false premise (p=0), the reasoning, i.e. the implication, is true regardless of the conclusion that we arrive at (q=0 or q=1). Finally, another characteristic feature of the classical logic is that the property of symmetry does not hold i.e. the truth value of the implication p⇒q generally has a different value from the truth value of the implication q⇒p. Indeed, from Table 2, we can see that the implication 1⇒0 has truth value of 0, while the symmetric equivalent implication 0⇒1, has a truth value of 1.

2 The Fuzzy Implication

The fuzzy implication assigns a truth value J(x,y) to the fuzzy proposition "If p then q" for every truth value x, y of the fuzzy propositions p, q. It is a function of the form $J:[0,1]\times[0,1]\rightarrow[0,1]$ which satisfies the following nine conditions, every one of which does not contain symmetry [2], [3], [4], [5]:

1. $\forall x,y,z\in[0,1],\ \ x\le z\Rightarrow J(x,y)\ge J(z,y)$,
2. $\forall x,y,z\in[0,1],\ \ y\le z\Rightarrow J(x,y)\le J(z,y)$,
3. $\forall y\in[0,1],\ \ J(0,y)=1$,
4. $\forall z\in[0,1],\ \ J(1,z)=1$,
5. $\forall x\in[0,1],\ \ J(x,x)=1$,
6. $\forall x,y,z\in[0,1],\ \ J(x,J(y,z)=J(y,J(x,z))$,
7. $\forall x,y\in[0,1],\ \ J(x,y)=1$ when $x\le y$,
8. $\forall x,y,z\in[0,1],\ \ J(x,y)=J(z(y),z(x))$, where z is a fuzzy complement,
9. J is a continuous function.

In many applications, i.e. in the fuzzy inference system of MATLAB (such an application can be found in [6]), the classic forms of implication, min (Mamdani) [7] and the prod (Larsen) [8] are used as first choices, where:

$$J_{Mamdani}(x,y)=\min\{x,\ y\} \tag{1}$$

$$J_{Larsen}(x,y)=x\cdot y \tag{2}$$

These implications are symmetric since x⇒y= y⇒x. These symmetric implications are being called engineering implications, because they are widely used in the field of engineering, where the cause and effect are often confused, hence the symmetry is acceptable. Apart from the above implications, other asymmetric implications have been proposed, they are (in curly brackets {} the axioms which are satisfied by every fuzzy implication), [3]:

$$J_{Kleene-Dienes}(x,y)=\max\{1-x,\ y\} \tag{3}$$
{1, 2, 3, 4, 6, 8, 9}

$$J_{Zadeh}(x,y)=\max(\min(x,y),1-x) \tag{4}$$
{1, 2, 3, 4, 9}

$$J_{Lukasiewicz}(x,y)=\min(1-x+y,1) \tag{5}$$
{1, 2, 3, 4, 5, 6, 7, 8, 9}

$$J_{Reichenbach}(x,y)=1-x+x\cdot y \tag{6}$$
{1, 2, 3, 4, 6, 8, 9}

We will apply the above symmetric and asymmetric fuzzy implications to a real data set from published statistical records, with the aim of evaluating the quality of the fuzzy implications. The existence of observations allows us to assume that in the ideal fuzzy inference system, the truth value of the implication x ⇒y *has to be equal to 1*, since the values of x and also of y are related to observations, that is they have to do with verified relations of assumed cause and implied effect. Starting from this finding, the evaluation of the fuzzy implications will be based on the deviation of the truth values of each implication from 1.

We must underline that a theoretical approach for the selection and the evaluation of the appropriate fuzzy implication, which was based on the Law of Modus Ponens and Modus Tollens, has been developed in the past but did not make use of statistical data, [10], [11], [12], [13], [14]. The current proposed methodology makes use of statistical data.

3 An Algorithm for the Evaluation of the Fuzzy Implication

3.1 The case study description

In the aviation industry, a high correlation has been established between the economic growths of a region, which is usually measured through the Gross Domestic Product (GDP), and the number of air trips made by its residents. There is a plethora of published work which confirms this correlation, [15], [16], [17], [18], [19], [20], [21], [22], [23].

Based on figures published by the World Investment Bank, [24], we give in Table 3 the per capita GDP for 2013 and the number of air trips per 1,000 residents in the year 2013 in 98 countries, which account for more than 80% of the GDP and the total air passenger demand worldwide.

3.2 Fuzzification of data

Considering the classification of countries by per capita GDP, proposed by the World Bank and other international organizations, and of the fact that the data in Table 3 relate to purchasing power parity, we form the fuzzy set *"Per capita GDP"* (GDP) and we divided it into *"Low per capita GDP"*, *"Medium per capita GDP"* and *"High per capita GDP"* with the help of triangular and trapezoidal fuzzy numbers (Figure 1). In the same manner, we described the fuzzy set *"Frequency of air trips"* (AT).

Table 3: Per capita GDP in US $ (Purchasing Power Parity, constant 2011 prices)
and air trips per 10,000 inhabitants for various countries worldwide

Country	Per capita GDP (US$)	Air trips per 10,000 inhabitants	Country	Per capita GDP (US$)	Air trips per 10,000 inhabitants	Country	Per capita GDP (US$)	Air trips per 10,000 inhabitants
Afghanistan	1,884	39.7	Greece	24,540	776.2	Paraguay	7,833	104.6
Albania	10,405	312.2	Guyana	6,336	240.1	Philippines	6,326	297.9
Algeria	12,893	115.7	Honduras	4,445	52.6	Portugal	25,596	1133.7
Australia	42,831	3064.4	Hong Kong	51,509	4759.1	Russian Fed.	23,564	459.8
Austria	44,376	1784.0	Hungary	22,914	964.0	Rwanda	1,426	45.8
Bahrain	42,428	3372.0	India	5,238	60.2	Saudi Arabia	52,068	1010.0
Bangladesh	2,853	13.3	Indonesia	9,254	340.6	Senegal	2,170	35.0
Benin	1,733	13.3	Kazakhstan	22,467	384.7	Serbia	12,893	184.4
Bhutan	7,167	288.3	Kenya	2,705	100.9	Singapore	76,237	5659.2
Bolivia	5,934	183.6	Kuwait	84,188	1004.1	South Africa	12,106	319.2
Brazil	14,555	478.7	Kyrgyz Rep.	3,110	91.9	South Asia	4,870	55.1
Brunei	69,474	2881.3	Lao PDR	4,667	170.0	Spain	31,596	1081.3
Burkina Faso	1,582	6.3	Lebanon	16,623	437.4	Sri Lanka	9,426	234.0
Cambodia	2,944	50.9	Libya	20,371	404.5	Sudan	3,265	14.7
Cameroon	2,739	12.9	Lithuania	24,483	353.5	Suriname	15,556	480.4
Canada	41,894	2034.4	Luxembourg	87,737	1633.9	Switzerland	54,697	3337.8
Chad	2,022	2.6	Madagascar	1,369	23.5	Tajikistan	2,432	76.6
Chile	21,714	783.2	Mali	1,589	2.2	Tanzania	1,718	27.9
China	11,525	259.9	Malta	28,828	3788.0	Togo	1,346	123.4
Congo, D.R.	783	2.8	Mauritania	2,945	79.5	Trinidad	29,469	1975.0
Congo, Rep.	5,680	164.4	Mexico	16,291	357.2	Turkey	18,660	992.3
Cote Ivoire	3,107	21.3	Moldova	4,521	156.7	Uganda	1,368	4.8
Croatia	20,063	403.7	Mongolia	9,132	224.9	Ukraine	8,508	118.2
Cyprus	27,394	1061.4	Morocco	6,967	203.4	UAE	57,045	7403.2
Egypt	10,733	120.8	Mozambique	1,070	24.4	UK	37,017	1845.7
Estonia	25,132	506.2	Namibia	9,276	222.7	USA	51,340	2350.6
Ethiopia	1,336	60.2	Nepal	2,173	25.2	Uzbekistan	5,002	85.0
Finland	38,846	1966.6	Netherlands	44,945	1978.7	Venezuela	17,615	350.7
France	37,154	1010.7	New Zealand	32,808	3068.2	Vietnam	5,125	203.8
Gambia	1,608	79.4	Niger	887	4.9	Yemen, Rep.	3,832	51.4
Georgia	6,946	38.3	Nigeria	5,423	21.6	Zambia	3,800	10.5
Germany	43,207	1302.6	Oman	42,649	1375.0	Zimbabwe	1,773	23.6
Ghana	3,864	31.5	Pakistan	4,454	42.8			

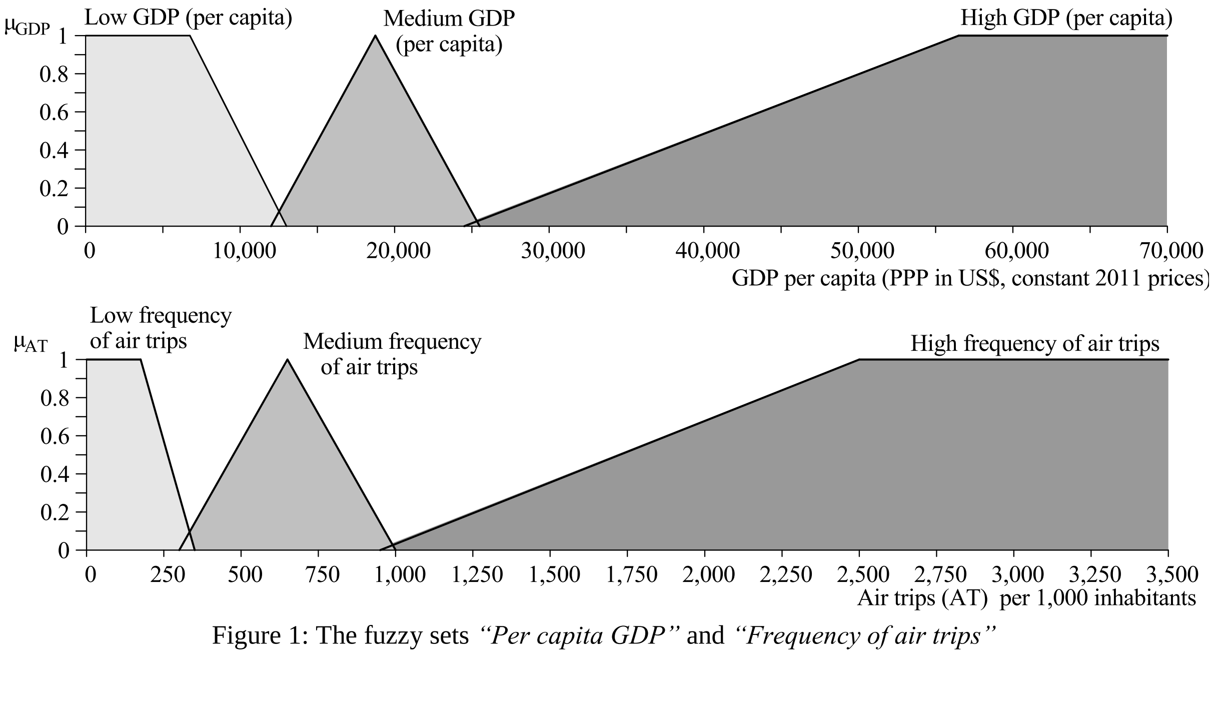

Figure 1: The fuzzy sets *"Per capita GDP"* and *"Frequency of air trips"*

The membership function of the trapezoidal fuzzy number *"Low per capita GDP"* is:

$$\mu_{\text{low GDP}}(x)=\begin{cases}1 & \text{for } 0\le x\le 6750\\ \dfrac{13000-x}{6250} & \text{for } 6750\le x\le 13000\\ 0 & \text{otherwise}\end{cases}\qquad(7)$$

the membership function for the triangular fuzzy number *"Medium per capita GDP"* is:

$$\mu_{\text{med. GDP}}(x)=\begin{cases}\dfrac{x-12000}{6750} & \text{for } 12000\le x\le 18750\\ \dfrac{25500-x}{6750} & \text{for } 18750\le x\le 25500\\ 0 & \text{otherwise}\end{cases}\qquad(8)$$

and the membership function of the trapezoidal fuzzy number *"High per capita GDP"* is:

$$\mu_{\text{high GDP}}(x)=\begin{cases}1 & \text{for } x\ge 56,000\\ \dfrac{x-24500}{32000} & \text{for } 24500\le x\le 56500\\ 0 & \text{otherwise}\end{cases}\qquad(9)$$

As for the frequency of air trips, the membership function of the trapezoidal fuzzy number *"Low frequency of air trips"* is:

$$\mu_{\text{low freq. trips}}(x)=\begin{cases}1 & \text{for } 0\le x\le 175\\ \dfrac{350-x}{175} & \text{for } 175\le x\le 350\\ 0 & \text{otherwise}\end{cases}\qquad(10)$$

the membership function of the triangular fuzzy number *"Medium frequency of air trips"* is:

$$\mu_{\text{med. freq. trips}}(x)=\begin{cases}\dfrac{x-300}{350} & \text{for } 300\le x\le 650\\ \dfrac{1000-x}{350} & \text{for } 650\le x\le 1000\\ 0 & \text{otherwise}\end{cases}\qquad(11)$$

while the membership function of the trapezoidal fuzzy number *"High frequency of air trips"* is:

$$\mu_{\text{high freq. trips}}(x)=\begin{cases}1 & \text{for } x\ge 2500\\ \dfrac{x-950}{1550} & \text{for } 950\le x\le 2500\\ 0 & \text{otherwise}\end{cases}\qquad(12)$$

3.3 Truth values of the data in the fuzzy sets

From the data of Table 3, and by using the equations (7) to (12), we get the truth values of the data in the fuzzy sets *"Per capita GDP"* and *"Frequency of air trips"*. In Table 4 we give some illustrative truth values for the *"Medium per capita GDP"* and the *"High per capita GDP"* as well as the corresponding truth values for the *"Medium frequency of air trips"* and the *"High frequency of air trips"* (the countries are classified by the size of their per capita GDP for the year 2013).

3.4 Truth values of the fuzzy implications

Using the equations (1) to (6) we determine the truth values of the fuzzy implications *"If low per capita GDP ⇒Low frequency of air trips"*, *"If medium per capita GDP ⇒Medium frequency of air trips"* and *"If high per capita GDP ⇒ High frequency of air trips"*. Some illustrative truth values of the various forms of fuzzy implications are given in Table 5.

However, as stated in Section 2, all the fuzzy implications refer to real observations and theoretically they should give truth values equal to 1. Therefore, we can evaluate the suitability of the six implications through the deviation of the real truth value of every implication from the theoretical value which is equal to 1. The calculated (sum of squared) deviations of the truth values of various implications are:

$$\sigma_{J_{\text{Mamdani}}}=5.12 \qquad \sigma_{J_{\text{Larsen}}}=5.61$$
$$\sigma_{J_{\text{Kleene-Dienes}}}=3.16 \qquad \sigma_{J_{\text{Zadeh}}}=3.35$$
$$\sigma_{J_{\text{Lukasiewicz}}}=2.44 \qquad \sigma_{J_{\text{Reichenbach}}}=2.69$$

The previous results show that the deviation of the $J_{\text{Lukasiewicz}}$ implication is 52.36% and 56.58% smaller respectively, when compared to the symmetric implications of J_{Mamdani} and J_{Larsen}.

4 Conclusions

This paper proposed a method for the selection of the appropriate fuzzy implication in a specific application from the airline industry, taking into account the advantages of using statistical data which correspond to real observations. This represents progress when compared to the arbitrary choice of fuzzy implications and in particular the use of symmetric implications of specialized software such as MATLAB.

An open question is to investigate the possibility that the truth values of an application, when using statistical data, does not have a value equal to the unit, but the truth values vary in an interval.

Table 4: Truth values of the data in the fuzzy sets *"Size of per capita GDP"* and *"Frequency of air trips"*

Country	Per capita GDP			Frequency of air trips		
	Value (in US$)	$\mu_{\text{medium GDP}}$	$\mu_{\text{high GDP}}$	Air trips per 10,000 inhabitants	$\mu_{\text{medium frequency}}$	$\mu_{\text{high frequency}}$
Brazil	14,555	1.000		478.72	0.000	
Suriname	15,556	1.000		480.40	0.000	
Mexico	16,291	1.000		357.23	0.000	
Lebanon	16,623	1.000		437.40	0.000	
Venezuela	17,615	1.000		350.66	0.000	
Turkey	18,660	1.000		992.27	0.000	
Croatia	20,063	1.000		403.67	0.000	
Libya	20,371	1.000		404.46	0.000	
Chile	21,714	1.000		783.18	0.000	
Kazakhstan	22,467	1.000		384.72	0.000	
Hungary	22,914	1.000		964.02	0.000	
Russian Fed.	23,564	1.000		459.83	0.000	
Lithuania	24,483	1.000		353.49	0.000	
Greece	24,540	1.000		776.19	0.000	0.000
Estonia	25,132	1.000		506.19	0.000	0.000
Portugal	25,596		1.000	1,133.71		0.119
Cyprus	27,394		1.000	1,061.38		0.072
Malta	28,828		1.000	3,788.03		1.000
Trinidad & Tobago	29,469		1.000	1,975.02		0.661
Spain	31,596		1.000	1,081.26		0.085
New Zealand	32,808		1.000	3,068.21		1.000
United Kingdom	37,017		1.000	1,845.71		0.578
France	37,154		1.000	1,010.68		0.039
Finland	38,846		1.000	1,966.57		0.656
Canada	41,894		1.000	2,034.42		0.700
Bahrain	42,428		1.000	3,371.98		1.000
Oman	42,649		1.000	1,375.03		0.274
Australia	42,831		1.000	3,064.44		1.000
Germany	43,207		1.000	1,302.58		0.227
Austria	44,376		1.000	1,784.04		0.538
Netherlands	44,945		1.000	1,978.66		0.664
United States	51,340		1.000	2,350.61		0.904
Hong Kong	51,509		1.000	4,759.11		1.000
Saudi Arabia	52,068		1.000	1,009.99		0.039
Switzerland	54,697		1.000	3,337.79		1.000
UAE	57,045		1.000	7,403.19		1.000
Brunei	69,474		1.000	2,881.28		1.000
Singapore	76,237		1.000	5,659.16		1.000
Kuwait	84,188		1.000	1,004.09		0.035
Luxembourg	87,737		1.000	1,633.92		0.441

Table 5: Truth values of the fuzzy implication *"If medium GDP per capita ⇒ Medium frequency of air trips"* and of the fuzzy implication *"If high GDP per capita ⇒ High frequency of air trips"*

Country	Medium GDP ⇒ Medium frequency of air trips						High GDP ⇒ High frequency of air trips					
	$J_{Mamdani}$	J_{Larsen}	$J_{Kleene…}$	J_{Zadeh}	$J_{Lukasiew.}$	$J_{Reichenb.}$	$J_{Mamdani}$	J_{Larsen}	$J_{Kleene…}$	J_{Zadeh}	$J_{Lukasiew.}$	$J_{Reichenb.}$
Brazil	0.000	0.000	0.386	0.651	0.143	0.143						
Suriname	0.000	0.000	0.235	0.531	0.235	0.235						
Mexico	0.000	0.000	0.700	0.803	0.404	0.404						
Lebanon	0.000	0.000	0.369	0.535	0.369	0.369						
Venezuela	0.000	0.000	0.731	0.774	0.692	0.692						
Turkey	0.000	0.000	0.956	0.957	0.956	0.956						
Croatia	0.000	0.000	0.495	0.580	0.495	0.495						
Libya	0.000	0.000	0.492	0.598	0.492	0.492						
Chile	0.000	0.000	0.193	0.426	0.145	0.193						
Kazakhstan	0.000	0.000	0.574	0.794	0.202	0.202						
Hungary	0.000	0.000	0.805	0.923	0.147	0.147						
Russian Fed.	0.000	0.000	0.509	0.755	0.082	0.082						
Lithuania	0.000	0.000	0.721	0.954	0.023	0.023						
Greece	0.000	0.000	0.736	0.826	0.020	0.020						
Estonia	0.000	0.000	0.894	0.937	0.003	0.003						
Portugal							0.000	0.000	0.933	0.992	0.001	0.001
Cyprus							0.000	0.000	0.861	0.987	0.008	0.008
Malta							0.000	0.000	0.748	0.748	0.000	0.018
Trinidad & Tobago							0.000	0.000	0.714	0.805	0.024	0.024
Spain							0.000	0.000	0.838	0.963	0.049	0.049
New Zealand							0.000	0.000	0.548	0.548	0.000	0.067
United Kingdom							0.000	0.000	0.371	0.599	0.153	0.153
France							0.000	0.000	0.923	0.969	0.156	0.156
Finland							0.000	0.000	0.304	0.498	0.118	0.201
Canada							0.000	0.000	0.208	0.384	0.090	0.208
Bahrain							0.000	0.000	0.193	0.193	0.000	0.193
Oman							0.000	0.000	0.527	0.713	0.322	0.322
Australia							0.000	0.000	0.182	0.182	0.000	0.182
Germany							0.000	0.000	0.597	0.752	0.342	0.342
Austria							0.000	0.000	0.213	0.443	0.213	0.213
Netherlands							0.000	0.000	0.130	0.332	0.113	0.130
United States							0.000	0.000	0.026	0.059	0.009	0.026
Hong Kong							0.000	0.000	0.024	0.024	0.000	0.024
Saudi Arabia							0.000	0.000	0.924	0.934	0.742	0.742
Switzerland							0.000	0.000	0.003	0.003	0.000	0.003
UAE							0.000	0.000	0.000	0.000	0.000	0.000
Brunei							0.000	0.000	0.000	0.000	0.000	0.000
Singapore							0.000	0.000	0.000	0.000	0.000	0.000
Kuwait							0.000	0.000	0.931	0.931	0.931	0.931
Luxembourg							0.000	0.000	0.312	0.312	0.312	0.312

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