Design of Stable IIR filters with prescribed flatness and approximately linear phase

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Abstract: This paper presents a design method of infinite impulse response (IIR) filters with prescribed flatness and approximately linear phase characteristics using quadratic programming (QP). It is utilized in this paper for the design of Chebyshev type, inverse Chebyshev type filters, and simultaneous Chebyshev type filters with the prescribed flatness in passband and stopband. In the proposed method, the flatness condition in stopband is preincorporated into the transfer function. Then, the flatness condition in passband and the filter's stability condition are, respectively, added to the QP problem as the linear matrix equality and linear matrix inequality constraints. As a result, the proposed method can easy design these three types of filter by only change of the design parameters. The effectiveness of the proposed design method is illustrated with some examples.

Key–Words: IIR filter, Flat magnitude, Flat group delay, Equiripple characteristics, Quadratic programming.

1 Introduction

Digital filters are classified into finite impulse response (FIR) and infinite impulse response (IIR) filters. FIR filters are always stable, and able to realize exactly linear phase characteristics. In contrast, IIR filters are not always stable, and cannot realize exactly linear phase characteristics. However, IIR filters are useful for high-speed processing, and IIR filters with lower order can be realized that are comparable to the amplitude characteristics of FIR filters. Therefore, several design methods for stable IIR filters with an approximately linear phase characteristic have been proposed [6]-[12].

Chebyshev type and inverse Chebyshev type filters [2]-[5] have an equiripple characteristic in either the passband or stopband, and a flat characteristic in the other band. These filters are effective for suppressing the ringing, and are well used in the fields of biosignal measurement and image processing. Moreover, magnitude flatness and multiple zeros are desirable in designing sample rate converters in order to suppress the alias components and the design of wavelet basis [13]. However, there is no a unified design method which can treat these three types of filters.

In [2] and [3], the design methods based on the Remez algorithm have been proposed for the Chebyshev type and inverse Chebyshev type IIR filters with approximately linear phase characteristics. These methods can design the filters with small computational complexity. However, the filters that can be designed using these methods are restricted greatly because of a condition imposed on setting the initial value. Moreover, the filters designed by these methods are not always stable. By using the linear semi-infinite programming and the extended positive realness, the design method of stable inverse Chebyshev type IIR filters with an approximately linear phase characteristic have been proposed [4]. However, the problem size depends on the number of the discrete frequency points in this method.

In this paper, a simple design method based on quadratic programming (QP) is proposed for the design of stable IIR filters with prescribed flatness and approximately linear phase characteristics. In the proposed method, the flat characteristics in stopband are realized first by placing multiple zeros in the stopband. Then, the frequency characteristics are approximated under the weighted least square criterion, by using the transfer function with the stopband flatness. The flatness condition in the magnitude and group delay in passband and the stability condition based on the extended positive realness [6] are added to the constraints of the QP problem by expressing as the linear matrix equality and linear matrix inequality constraints. The equiripple characteristics are met by adjusting the weighting function using the modified Lawson's method [14] and then solving iteratively the QP problem. The proposed method can easy design not only the Chebyshev type and inverse Chebyshev type filters but also simultaneous Chebyshev type filters with the prescribed flatness in passband and stopband. The effectiveness of the proposed method is verified through some design examples.

2 IIR digital filters and Flatness conditions

The frequency response $H(e^{j\omega})$ of an IIR digital filter is defined as

$$H(e^{j\omega}) = \frac{A(e^{j\omega})}{B(e^{j\omega})} = \frac{\sum_{n=0}^{N} a_n e^{-jn\omega}}{\sum_{m=0}^{M} b_m e^{-jm\omega}} \tag{1}$$

where N and M are the orders of the numerator and denominator, respectively. The filter coefficients a_n and b_m are real, and $b_0 = 1$ in general. The desired frequency response $H_d(e^{j\omega})$ of a lowpass filter can be expressed as

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\tau_d\omega} & (0 \le \omega \le \omega_p) \\ 0 & (\omega_s \le \omega \le \pi) \end{cases} \tag{2}$$

where τ_d is a desired group delay in the passband and ω_p and ω_s are, respectively, passband and stopband edge angular frequencies. Then, the flatness conditions of the magnitude and group delay in the passband are given as follows [3]:

$$\left.\frac{\partial^i |H(e^{j\omega})|}{\partial \omega^i}\right|_{\omega=0} = \begin{cases} 1 & (i=0) \\ 0 & (i=1,2,\ldots,K_p-1) \end{cases} \tag{3}$$

$$\left.\frac{\partial^i \tau(\omega)}{\partial \omega^i}\right|_{\omega=0} = \begin{cases} \tau_d & (i=0) \\ 0 & (i=1,2,\ldots,K_p-2) \end{cases} \tag{4}$$

where K_p is a parameter expressing the flatness in the passband. The magnitude flatness condition in the stopband is

$$\left.\frac{\partial^i |H(e^{j\omega})|}{\partial \omega^i}\right|_{\omega=\pi} = 0 \qquad (i=0,1,\ldots,K_s-1) \tag{5}$$

where K_s is a parameter expressing the flatness in the stopband.

3 Proposed Algorithm

3.1 Flatness condition in stopband

First, we consider the flatness condition in the stopband. Let $\hat{H}(e^{j\omega})$ be a noncausal shifted version of $H(e^{j\omega})$;

$$\hat{H}(e^{j\omega}) = H(e^{j\omega})e^{j\tau_d\omega} = \frac{\sum_{n=0}^{N} a_n e^{-j(n-\tau_d)\omega}}{\sum_{m=0}^{M} b_m e^{-jm\omega}} \tag{6}$$

Then, the flatness condition in eq. (5) becomes

$$\left.\frac{\partial^i |\hat{H}(e^{j\omega})|}{\partial \omega^i}\right|_{\omega=\pi} = 0 \qquad (i=0,1,\ldots,K_s-1). \tag{7}$$

In order to meet the flatness condition of eq. (7), it is necessary to place K_s multiple zeros at $\omega = \pi$. Hence, the frequency response $\tilde{H}(e^{j\omega})$ can be expressed as follows [2].

$$\tilde{H}(e^{j\omega}) = \frac{\tilde{A}(e^{j\omega})}{B(e^{j\omega})} = \frac{(1+e^{-j\omega})^{K_s} \sum_{n=0}^{N-K_s} c_n e^{-j(n-\tau_d)\omega}}{1 + \sum_{m=1}^{M} b_m e^{-jm\omega}}. \tag{8}$$

3.2 Flatness condition in passband

With eq. (8), the flatness conditions in eqs. (3) and (4) become

$$\left.\frac{\partial^i \tilde{H}(e^{j\omega})}{\partial \omega^i}\right|_{\omega=0} = \begin{cases} 1 & (i=0) \\ 0 & (i=1,2,\ldots,K_p-1) \end{cases}. \tag{9}$$

Eq. (9) is the same as follows.

$$\left.\frac{\partial^i \tilde{A}(e^{j\omega})}{\partial \omega^i}\right|_{\omega=0} = \left.\frac{\partial^i B(e^{j\omega})}{\partial \omega^i}\right|_{\omega=0} \quad (i=0,1,\cdots,K_p-1). \tag{10}$$

Consequently, we can get the following linear equation in matrix form:

$$\boldsymbol{U}\boldsymbol{h} = \boldsymbol{V} \tag{11}$$

where

$$\boldsymbol{h} = [c_0, \ldots, c_{N-K_s}, b_1, \ldots, b_M]^T, \tag{12}$$

$$\boldsymbol{V} = [1, 0, \ldots, 0]^T, \tag{13}$$

$$\boldsymbol{U} = \left[\begin{array}{cc} \boldsymbol{U_1} & \boldsymbol{U_2} \end{array} \right], \tag{14}$$

$$\boldsymbol{U_1} = \left[\begin{array}{ccc} u(0,0) & \ldots & u(0, N-K_s) \\ \vdots & \ddots & \vdots \\ u(K_p-1, 0) & \ldots & u(K_p, N-K_s) \end{array} \right], \tag{15}$$

$$\boldsymbol{U_2} = \left[\begin{array}{ccc} -1^0 & \ldots & -M^0 \\ \vdots & \ddots & \vdots \\ -1^{K_p-1} & \ldots & -M^{K_p-1} \end{array} \right], \tag{16}$$

$$u(p,q) = \sum_{k=0}^{p} \sum_{r=0}^{K_s} {}_pC_k {}_{K_s}C_r r^{p-k} (q-\tau_d)^k, \tag{17}$$

and $(\cdot)^T$ denotes the transpose of $(\cdot)$.

3.3 Formulating the design as a QP problem

If the frequency response of eq. (8) is used, it is found that the flatness characteristic at $\omega = \pi$ can easily be realized.

Here, let $\tilde{H}_d(e^{j\omega})$ be the desired magnitude response of a lowpass filter, i.e.,

$$\tilde{H}_d(e^{j\omega}) = \begin{cases} 1 & (0 \le \omega \le \omega_p) \\ 0 & (\omega_s \le \omega \le \pi) \end{cases}. \tag{18}$$

Using eqs. (8) and (18), the weighted least squares design problem is

$$\min_{\boldsymbol{c},\boldsymbol{b}} J = \sum_{l=1}^{L} W(\omega_l) |\tilde{H}(e^{j\omega_l}) - \tilde{H}_d(e^{j\omega_l})|^2 \tag{19}$$

where L is the total number of grid points in the passband and stopband, $W(\omega_l)$ is the weighting function, and $\omega_l (l = 1, \ldots, L)$ are the discrete frequency points used in the calculation. However, it is difficult to solve eq. (19) directly because $\tilde{H}(e^{j\omega_l})$ is a rational function. Thus, we use the following iterative design formula:

$$\min_{\boldsymbol{c},\boldsymbol{b}} J = \sum_{l=1}^{L} \frac{W(\omega_l) |\tilde{A}(e^{j\omega_l}) - \tilde{H}_d(e^{j\omega_l}) B(e^{j\omega_l})|^2}{|B_{k-1}(e^{j\omega_l})|^2} \tag{20}$$

where k is the number of the iterations.

After some manipulation, eq. (20) can be formulated as the following QP problem:

$$\begin{aligned} \min_{\boldsymbol{h}_k} \; & \boldsymbol{h}_k^T \left(\mathrm{Re}(\boldsymbol{P}^T) \boldsymbol{W} \mathrm{Re}(\boldsymbol{P}) + \mathrm{Im}(\boldsymbol{P}^T) \boldsymbol{W} \mathrm{Im}(\boldsymbol{P}) \right) \boldsymbol{h}_k \\ & -2 \left(\mathrm{Re}(\boldsymbol{Q}^T) \boldsymbol{W} \mathrm{Re}(\boldsymbol{P}) + \mathrm{Im}(\boldsymbol{Q}^T) \boldsymbol{W} \mathrm{Im}(\boldsymbol{P}) \right) \boldsymbol{h}_k \end{aligned} \tag{21}$$

where

$$\boldsymbol{P} = diag(\boldsymbol{G}) \left[\begin{array}{ccc} e^{-j0\omega_1} & \ldots & e^{-j(N-K_s-\tau_d)\omega_1} \\ \vdots & \ddots & \vdots \\ e^{-j0\omega_L} & \ldots & e^{-j(N-K_s-\tau_d)\omega_L} \end{array} \right], \tag{22}$$

$$\boldsymbol{Q} = diag(\boldsymbol{d}) \left[\begin{array}{ccc} e^{-j\omega_1} & \ldots & e^{-jM\omega_1} \\ \vdots & \ddots & \vdots \\ e^{-j\omega_L} & \ldots & e^{-jM\omega_L} \end{array} \right], \tag{23}$$

$$\boldsymbol{G} = [(1+e^{-j(\omega_1)})^{K_s}, \ldots, (1+e^{-j(\omega_L)})^{K_s}], \tag{24}$$

$$\boldsymbol{d} = [H_d(e^{j\omega_1}), \ldots, H_d(e^{j\omega_L})], \tag{25}$$

$$\boldsymbol{W} = diag\left(\left[\frac{W(\omega_1)}{|B_{k-1}(\omega_1)|^2}, \ldots, \frac{W(\omega_L)}{|B_{k-1}(\omega_L)|^2} \right] \right). \tag{26}$$

3.4 Update of the weighting function $W(\omega)$

It has been well known that the filters obtained under weighted least square criterion have a large magnitude ripple near the band edges. So in order to realize the equiripple characteristics in the passband or stopband or both, the weighting function used at every iteration is adjusted using the modified Lawson's method [14] and the QP problem is solved to obtain the coefficients. In this paper, the weighting function $W(\omega)$ in kth iteration step is updated as follows:

$$W_{k+1}(\omega) = \frac{W_k(\omega)\beta_k(\omega)}{\frac{1}{L}\sum_{l=1}^{L} W_k(\omega_l)\beta_k(\omega_l)} \tag{27}$$

where the envelope function $\beta_k(\omega)$ is given as the function of straight line formed by joining together all the extremal points of the same frequency band of interest on the error function which is expressed as

$$E_k(\omega) = \left| \tilde{H}(e^{j\omega}) - \tilde{H}_d(e^{j\omega}) \right|. \tag{28}$$

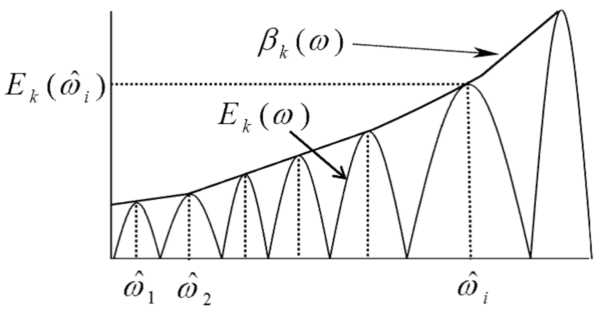


Figure 1: Example of envelope function

Using the extremal points $\hat{\omega}_i$ of $E_k(\omega)$, $\beta_k(\omega)$ can be calculated by

$$\beta_k(\omega) = \frac{\omega - \hat{\omega}_i}{\hat{\omega}_{i+1} - \hat{\omega}_i} E_k(\hat{\omega}_{i+1}) + \frac{\hat{\omega}_{i+1} - \omega}{\hat{\omega}_{i+1} - \hat{\omega}_i} E_k(\hat{\omega}_i) \quad \text{for } \hat{\omega}_\text{i} < \omega < \hat{\omega}_\text{i+1} \tag{29}$$

where $\hat{\omega}_i$ denotes the ith extremal frequency of the the error function $E_k(\omega)$. An example of $\beta_k(\omega)$ is depicted in Fig. 1.

3.5 Stability Constraint

A stability condition based on a positive realness in [1] is given by

$$\text{Re}\left\{B(e^{j\omega})\right\} \geq \epsilon_p \qquad \forall\omega[0:\pi], \tag{30}$$

where ϵ_p is a positive small value. This stability condition has been applied to many design methods as in [7]-[11]. However, the use of eq. (30) may exclude the candidate for the transfer function with excellent performance because this condition is a sufficient condition to assure the stability and is often too restrictive. Moreover, there is a disadvantage that it is difficult to prespecify the stability margin which is the distance between the maximum pole radius and the unit circle. In [6], an iterative method for the stability guarantee based on the positive realness was proposed in order to get a better transfer function. In this method, a stability condition is given by

$$\text{Re}\left\{B(e^{j\omega})\right\} \geq \delta \qquad \forall\omega[0:\pi], \tag{31}$$

where $\delta < 1$. If the maximum pole radius P_m of the filter obtained using a given δ is greater than prescribed maximum allowable pole radius P_R, δ is updated and redesign is carried out using the updated δ. The update of δ and the redesign are repeated until satisfy $|P_m - P_R| \leq \epsilon_R$, where ϵ_R is a positive small value. The update procedure of δ is described in next subsection "Design Procedure".

Using the discrete angular frequency $\omega_i (i = 1, ..., R)$, eq. (31) can be expressed as the linear matrix inequality

$$\boldsymbol{\Gamma h} \geq \boldsymbol{\lambda} \tag{32}$$

where

$$\boldsymbol{\Gamma} = \begin{bmatrix} 0 & \ldots & 0 & \cos(\omega_1) & \ldots & \cos(M\omega_1) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & 0 & \cos(\omega_R) & \ldots & \cos(M\omega_R) \end{bmatrix},$$

$$\boldsymbol{\lambda} = [\delta - 1, \cdots, \delta - 1]^T. \tag{33}$$

Thus, the design problem in which the flatness condition and stability constraint were considered becomes a standard QP problem below:

$$\min_{\boldsymbol{h}_k} \ \boldsymbol{h}_k^T \left(\text{Re}(\boldsymbol{P}^T)\boldsymbol{W}_k\text{Re}(\boldsymbol{P}) + \text{Im}(\boldsymbol{P}^T)\boldsymbol{W}_k\text{Im}(\boldsymbol{P})\right)\boldsymbol{h}_k$$
$$-2\left(\text{Re}(\boldsymbol{Q}^T)\boldsymbol{W}_k\text{Re}(\boldsymbol{P}) + \text{Im}(\boldsymbol{Q}^T)\boldsymbol{W}_k\text{Im}(\boldsymbol{P})\right)\boldsymbol{h}_k$$

$$\text{sub. to} \quad \boldsymbol{\Gamma h}_k \geq \boldsymbol{\lambda}$$
$$\boldsymbol{U h}_k = \boldsymbol{V}. \tag{34}$$

This problem can be solved using a powerful QP tool, such as *quadprog* in MATLAB.

3.6 Design Procedure

The design procedure of the proposed method is summarized as follows.

Step 0: Set the desired magnitude response $\tilde{H}_d(\omega)$, group delay response τ_d filter order N and M, flatness K_p and K_s, passband edge ω_p, stopband edge ω_s, weighing function $W(\omega)$, initial value of δ, number of grid points L and R.

Step 1: Solve the QP problem in eq. (34) to obtain the filter coefficient $\boldsymbol{h}_k$.

Step 2: If $\dfrac{\text{sum}(|\boldsymbol{h}_k - \boldsymbol{h}_{k-1}|)}{\text{sum}(|\boldsymbol{h}_k|)} \leq \epsilon$ and $P_m < P_R$, stop, if $\dfrac{\text{sum}(|\boldsymbol{h}_k - \boldsymbol{h}_{k-1}|)}{\text{sum}(|\boldsymbol{h}_k|)} \leq \epsilon$ and $P_m > P_R$, go to step 4; otherwise, go to step 3.

Step 3: Update the weighting function $W(\omega)$ using eqs. (27) - (29) and go back to Step 1.

Step 4: Calculate $\delta' = \min_{all\ \omega} \text{Re}\{B(\omega)\}$ from the obtained filter, and then set to $\delta_l = \delta'$ and $\delta_u = 1$.

Step 5: Calculate $\delta = (\delta_l + \delta_u)/2$.

Step 6: Solve the QP problem in eq. (34) using the updated δ.

Step 7: If $\dfrac{\text{sum}(|\boldsymbol{h}_k - \boldsymbol{h}_{k-1}|)}{\text{sum}(|\boldsymbol{h}_k|)} \leq \epsilon$ and $|P_m - P_R| \leq \epsilon_R$, stop, if $\dfrac{\text{sum}(|\boldsymbol{h}_k - \boldsymbol{h}_{k-1}|)}{\text{sum}(|\boldsymbol{h}_k|)} \leq \epsilon$ and $|P_m - P_R| > \epsilon_R$, go to Step 9; otherwise then go to Step 8.

Step 8: Update the weighting function $W(\omega)$ using eqs. (27) - (29) and go back to Step 6.

Step 9: If $P_m < P_R$, set to $\delta_u = \delta$, if $P_m > P_R$, set to $\delta_l = \delta$, and then go back to Step 5.

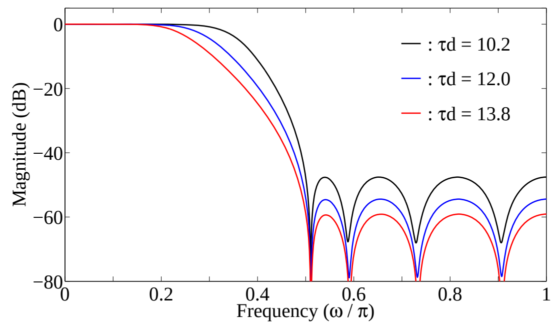
4 Design examples

In this section, the some design examples are given to illustrate the effectiveness of the proposed design method. In all the following examples, $R = 1000$, $\epsilon = 10^{-7}$, $\epsilon_R = 10^{-5}$, and the initial value of δ is -10^2. Moreover, "*quadprog*" function in MATLAB was used to solve the QP problem in eq. (34).

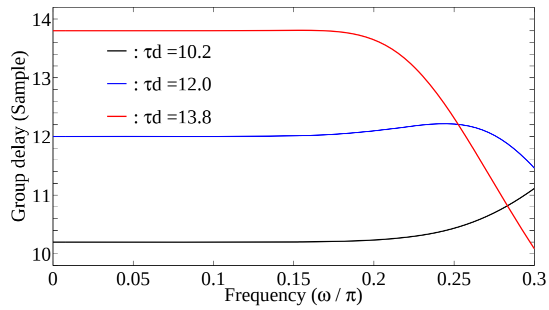
4.1 example 1

We first show the design examples of the inverse Chebyshev type IIR filters which have a flat characteristic in passband and an equiripple characteristic in stopband. The design specifications are as follows: $N = 12, M = 5, \omega_s = 0.50\pi, \tau_d = \{10.2, 12.0, 13.8\}, K_p = 10, K_s = 0, P_m = 1.0$. The total grid point L is 500 which is the sum of 0 point in the passband and 500 points in the stopband. The magnitude response and group delay response of the obtained filter are shown in Fig. 2. The numerical performance are shown in Table 1 with them obtained by the conventional method [3] based on Remez algorithm. From Fig. 2, it is confirmed that the magnitude and group delay responses both have a flat characteristic at $\omega = 0$ and the magnitude response in stopband is equiripple. From Table I, it is confirmed that the resulting filters by the proposed method have almost the same or better characteristics compared with them by the conventional method.

Next, we consider the following specifications: $N = 14, M = 9, \omega_s = 0.40\pi, \tau_d = 11.0, K_p = 9$. In [3], the condition that $K_p \geq M+1$ must be met in order to set the initial value. Therefore, the conventional method [3] cannot design this filter. The magnitude response and the group delay response of the filter obtained by the proposed method are shown in Fig. 3. It is confirmed from Fig. 3 that both of the magnitude and group delay responses of the obtained filter have a flat characteristic at $\omega = 0$ and the magnitude response in stopband is equiripple. The minimum stopband attenuation of the obtained filter is 51.45dB and the maximum pole radius is 0.827. Hence, the obtained filter is stable.



(a) Overall magnitude response

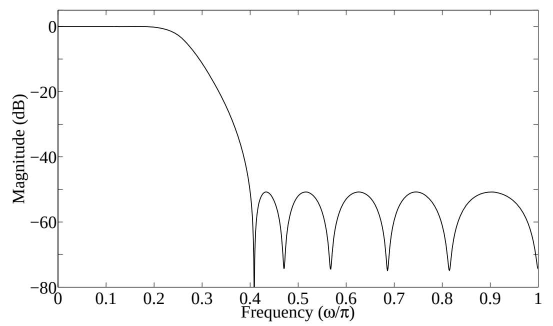


(b) Group delay response

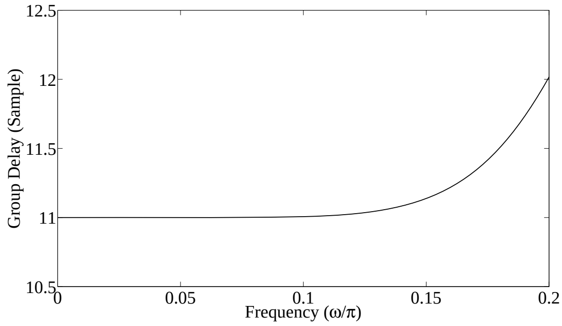
Figure 2: Inverse Chebyshev type IIR filters with $N = 12$ and $M = 5$ in example 1.

4.2 example 2

We show the design examples of the Chebyshev type IIR filters which have an equiripple characteristic in passband and a flat characteristic in stopband. The design specifications are as follows: $N = 15, M = 6, \omega_p = 0.30\pi, \tau_d = 12.0, K_p = 0, K_s = \{9, 10, 11\}, P_m = 1.0$. The total grid pont L is 500 which is the sum of 500 points in the passband and 0 point in the stopband. The magnitude response and group delay response of the obtained filter are shown in Fig. 4(a)-(c). The performance of each filter is shown in Table II with them obtained by the conventional method [2] based on Remez algorithm. In Ta-



(a) Overall magnitude response



(b) Group delay response

Figure 3: Inverse Chebyshev type IIR filter with $N = 14$ and $M = 9$ in example 1.

Table 1: Comparison with Ref.[3]

	τ_d	Minimum Stopband Attenuation [dB]	Maximum Pole Radius
Proposed	10.2	47.58	0.774
	12.0	54.45	0.732
	13.8	59.15	0.740
Ref. [3]	10.2	46.70	0.774
	12.0	53.62	0.732
	13.8	58.34	0.740

Table 2: Comparison with Ref.[2]

	K_s	R_p	G_{err}	P_{max}
Proposed	9	2.00×10^{-6}	1.07×10^{-4}	0.757
	10	2.65×10^{-5}	3.29×10^{-3}	0.805
	11	1.10×10^{-4}	3.23×10^{-2}	0.858
Ref. [2]	9	2.01×10^{-6}	1.11×10^{-4}	0.758
	10	2.65×10^{-5}	3.31×10^{-3}	0.806
	11	1.17×10^{-4}	3.24×10^{-2}	0.858

Table 3: Resulting filters in example 3

K_p	R_p	R_s [dB]	G_{err}	P_{max}
2	0.0261	31.28	1.494	0.8882
6	0.0297	30.14	1.382	0.8841
10	0.0169	25.56	1.654	0.8690

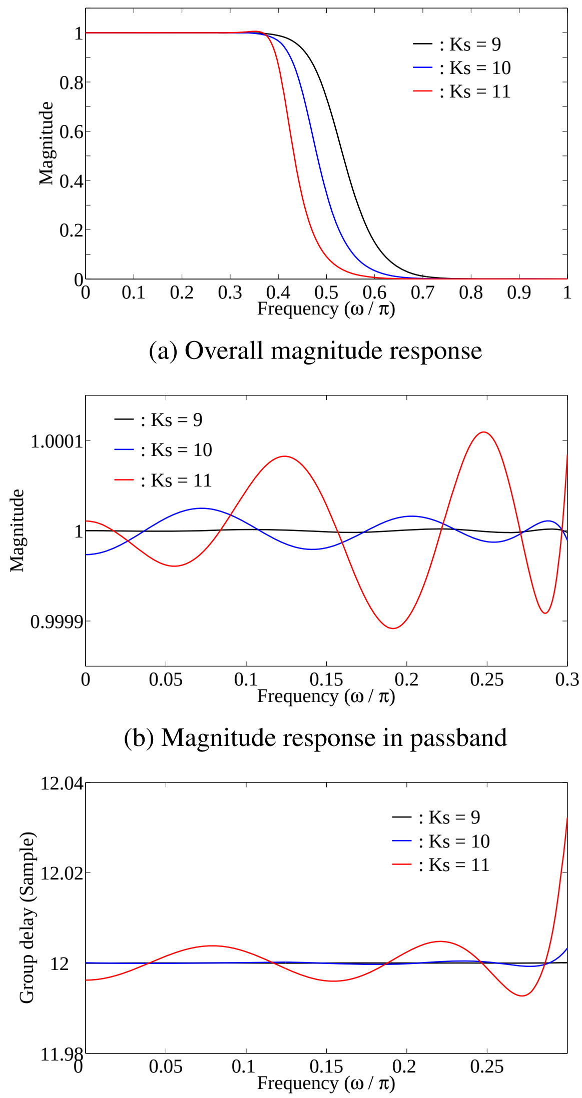
ble II, R_p is the maximum magnitude error in passband, G_{err} is the maximum group delay error in passband, and P_{max} is the maximum pole radius. From these figures, it is confirmed that the obtained filters have the equiripple characteristics in passband and the flat characteristics in stopband. From Table II, it is confirmed that the resulting filters by the proposed method have almost the same or better characteristics compared with them by the conventional method.

4.3 example 3

We show the design examples of the equiripple IIR filters with the flat characteristics at $\omega = 0$ and $\omega = \pi$. The design specifications are as follows: $N = 12, M = 6, \omega_p = 0.50\pi, \omega_s = 0.60\pi, \tau_d = 10.0, K_p = \{2, 6, 10\}$, $K_s = 2, P_m = 1.0$. The total grid pont L is 1000 which is the sum of 500 points in the passband and 500 points in the stopband. The magnitude response and group delay response of the obtained filter are shown in Fig. 5(a) - (c) and the complex magnitude error and pole-zero plot of the filter with $k_p = 6$ are depicted in Fig. 5(d) and (e). Moreover, the performance of each filter is shown in Table III. In Table III, R_p is the maximum magnitude error in passband,R_s is the minimum stopband attenuation, G_{err} is the maximum group delay error in passband, and P_{max} is the maximum pole radius. From these figures, it is confirmed that both the magnitude and group delay responses have a flat characteristic at $\omega = 0$ and $\omega = \pi$ and the equiripple characteristics are obtained in other interest region.

5 Conclusion

In this paper, a design method based on Quadratic programming has been proposed for approximately linear phase IIR filters with prescribed flatness in passband or stopband, or both. The flat stopband characteristics can be realized by placing multiple zeros in the stopband. Therefore, the flat condition in stopband is preincorporated into the transfer function. With



(a) Overall magnitude response

(b) Magnitude response in passband

(c) Group delay response

Figure 4: Chebyshev type IIR filters in example 2.

the transfer function, we formulated the approximation problem of the frequency characteristics as a QP problem with the linear matrix equality and linear matrix inequality constraints that are the flat condition in passband and the stability condition for the obtained filters. In the proposed method, the stable IIR filters, which have an equiripple response and prescribed flatness, are obtained by solving iteratively the QP problem. In the design examples, we showed that the proposed method can design the filters better than the conventional method and that the filters which cannot be designed by the conventional method could also be designed.

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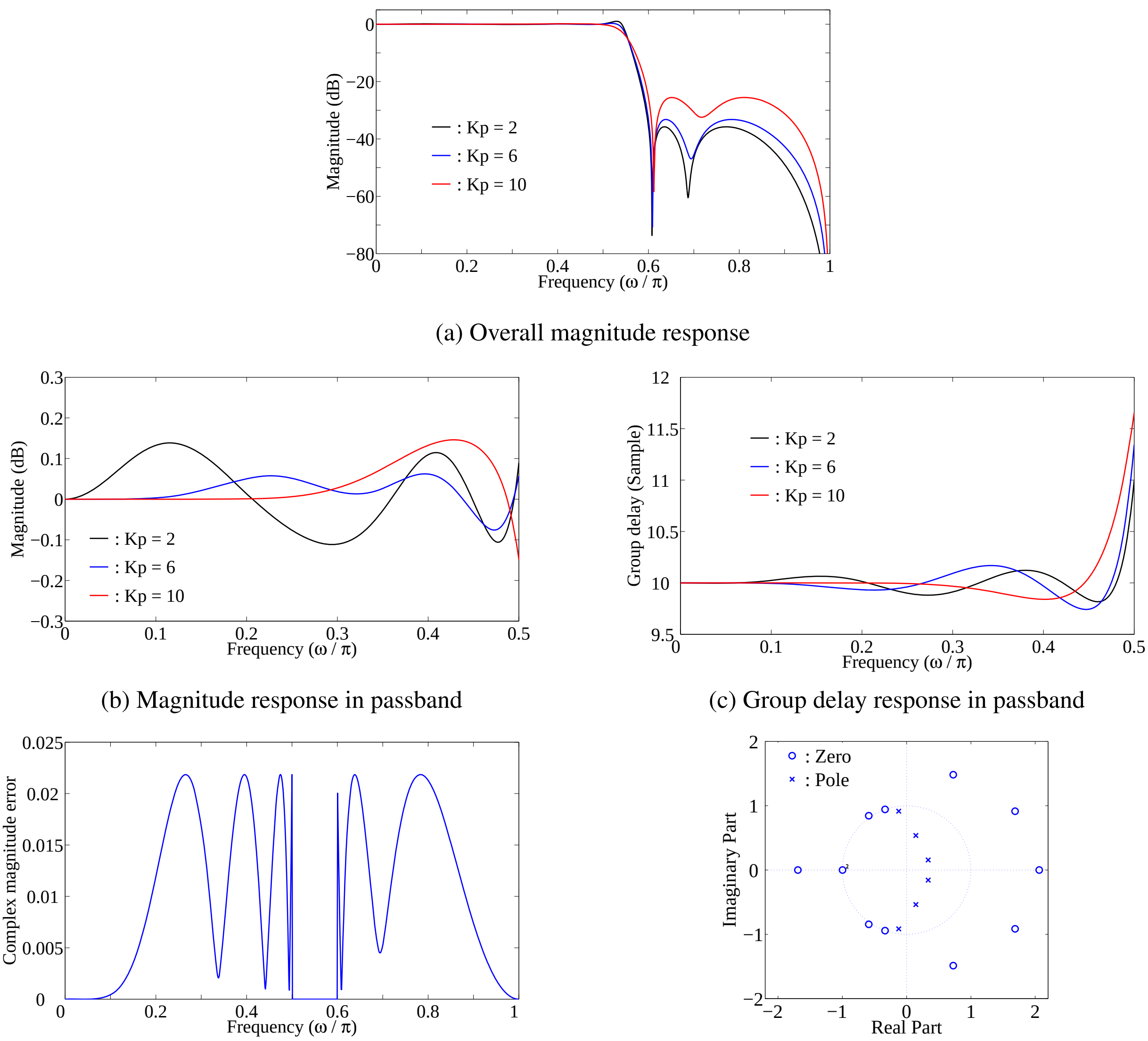

(a) Overall magnitude response

(b) Magnitude response in passband

(c) Group delay response in passband

(d) Complex magnitude error in passband ($k_p = 6$)

(e) Pole-zero plot ($k_p = 6$)

Figure 5: Simultaneous Chebyshev type IIR filters with flatness in example 3.

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