The swing-by effect in the Vesta-Magnya case. Single and multiple encounters.

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Abstract - It was verified that the asteroid Magnya has some physical and chemical characteristics similar to the bodies from Vesta family. However, astronomical observations revealed that Magnya is distant from these bodies. In the present work, we assumed that Magnya originated from Vesta and we try to justify its current distant orbital location taking into account the effects of close encounters between Magnya and Vesta. The methodology adopted involved an analytical approach considering the technique of the gravity assisted maneuver, also known as swing-by. We found that the energy variation achieved through a single swing-by between Vesta and Magnya are very small when compared to the variation that would be required to change the orbit of Magnya. The effects of multiple close encounters were also considered and discussed. We concluded that the possibility of multiple encounters is limited, and therefore, that Magnya should suffer other perturbations (such as resonances, collisions or close encounters with other bodies, for instance) that would provide the supposed change in its orbit.

Key-Words: Swing-by, close encounter, Vesta, Magnya.

1 Introduction.

In the Solar System there is a large concentration of asteroids located between the orbits of the planets Mars and Jupiter called the main asteroid belt. It is believed that collisions between large asteroids generate the families of asteroids currently observed, composed of bodies with close orbital elements (vicinity of the largest asteroid) and with similar physical and chemical characteristics, emphasizing the idea that the formation of a family occurs by disruption of a larger body [1],[2].

The asteroid Vesta is the only large asteroid of the main asteroid belt that presents a basalt surface, probably originated from ancient volcanic eruptions [3]. Analysis of images of the Hubble Space Telescope indicate the presence of calcium, magnesium and olivine [4]. There is also a large impact crater, which probably generated all the small bodies that compose the Vesta family. [5].

In the year 2000, it was verified that the asteroid Magnya has similar spectral characteristics to the bodies of the Vesta family, i.e., it also presents a basalt formation [6]. This was the first V-type asteroid observed outside the Vesta family. Actually, Magnya has an orbital location (semi-major axis, eccentricity, inclination) very far when compared with the other members of the Vesta family (see figure 1 and Table 1).

More recent studies show that there are many others V-types asteroids in the main belt found outside the Vesta family [7]. It was verified that asteroid Vesta can be the source of some of these bodies. In reference [8] was modeled the effects of close encounters with massive asteroids to estimate the scattering of small bodies initially placed in the vicinity of Vesta. In reference [9] they took into account the effects of nonlinear secular resonances and the Yarkovsky effect in order to explain the orbits of 956 Elisa and 809 Lundia (V-types asteroids outside the borders of the Vesta family). The reference [10] performed numerical integration of a thousand test bodies starting in the vicinity of Vesta in order to reproduce the dynamical evolution of the family. They discuss the characteristics of the dispersion of these bodies and the cases where Vesta seems to be the source. None of these cited works explain the location of Magnya with respect to the Vesta family.

The reference [11] presents a discussion on the origin of Magnya, where the hypothesis of having Vesta as the origin of Magnya is not completely discarded. Nevertheless, the difficulty to find a dynamical mechanism that relate the origin of Magnya to Vesta give rises to a new hypothesis which consider that Magnya is actually a fragment of another differentiated parent body, not Vesta, which has not been observed yet, as discussed in [10], [11] and [12].

The origin of Magnya is not a well-constrained problem. Thus, in this paper we considered that Magnya originated from Vesta, and we analyze the effects of a close approach between Vesta and Magnya, applying the technique of the gravity assist maneuver, also known as swing-by, in order to try to explain the orbital location of Magnya with respect to the bodies from Vesta family.

The structure of the paper is as follow. In section 2, the theory of the gravity assist maneuver and the mathematical description of the maneuver is presented. In the section 3, we discuss the methodology adopted. In section 4, the results are presented and discussed, and in section 5 is the final comments.

2 The gravity assist maneuver - or swing-by.

A large number of space missions have already implemented, or plan to apply, the technique of the gravity assist maneuver, or swing-by, to accomplish their goal. A successful example is the Voyager mission, which flew to the outer planets of the solar system through successive swing-bys with the planets visited [14]. Other examples of spatial missions that applied the gravity assist maneuver are the Cassini-Huygens mission [15], the Rosetta mission [16] and the New Horizons mission [17].

Due to its importance and large application, the swing-by has been under study by several researches. Some examples are shown below. Reference [18] shows a study of the impact of the initial energy in a close approach of a cloud of particles. In this paper a study is made to see the effects of a three-dimensional close approach between a planet and a cloud of particles. The objective is to learn the behavior of this cloud in terms of the variations of velocity, energy, angular momentum and inclination. The problem is solved by developing a numerical algorithm and applying it to a cloud of particles. This algorithm is based in an analytical description of the close approach maneuver in the three-dimensional space.

Similar application is shown in references [19] and [20]. In both references a study is made considering a Swing-by maneuver for a cloud of particles with several planets of the solar system. The idea is to study the importance of the mass of the celestial body in the results. The particles are assumed to be moving under the gravitational attraction of the two primaries (Sun and planet). The motion is planar for all the particles and the dynamics given by the "patched-conic" approximation is used. Reference [19] concentrates in studying the effects of the periapsis distance in this maneuver and reference [20] is devoted to the comparison between numerical and analytical methods to study this problem.

Following is the mathematical description and the discussion about the parameters involved in the swing-by. We follow the description found [21] that was based in [22].

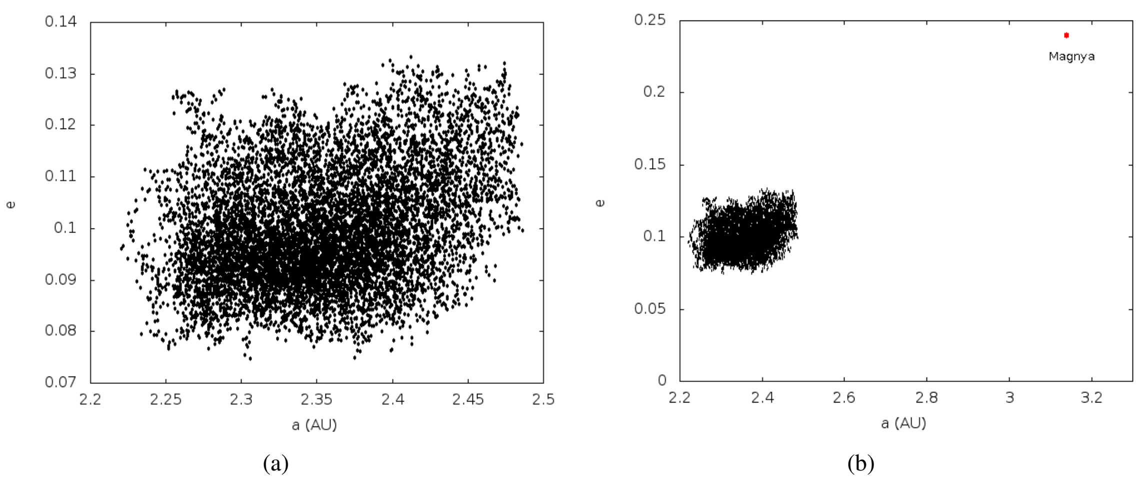

Figure 1: a) Proper elements (a×e) for 13800 bodies belonging to the Vesta family [13]. Vesta is at $a = 2.36$ AU with $e = 0.09$ (Table 1). b) Comparison between proper elements of these bodies with the proper elements of Magnya (red dot).

Table 1: Orbital and physical data for Vesta and Magnya.

	Vesta	Magnya
a (AU)	2.36	3.14
e	0.09	0.23
I	7.13°	16.9°
Mass (kg)	2.71×10^{20}	*
Diameter (km)	501	30

* not determined.

2.1 Mathematical description.

It is considered a system composed by three bodies: M_1, a massive body in the center of a Cartesian reference system; M_2, a smaller body in a keplerian orbit around M_1; and M_3, a body with infinitesimal mass, or a particle, that is also in a keplerian orbit around M_1. During its orbital evolution, M_3 suffers a close encounter with M_2, changing its initial orbit.

The phenomenon of change of orbit due to the encounter with a massive body is called gravity assist maneuver, or swing-by. The figure 2 is a representation of such maneuver, and illustrate some of the variables related to the maneuver. $\vec{V}_2$ is the velocity vector of the body M_2 relative to M_1. $\vec{V}_\infty^-$ and $\vec{V}_\infty^+$ are the velocity vector of M_3 relative to M_2, before and after the close encounter, respectively. $\vec{V}_i$ and $\vec{V}_0$ are the velocity vector of M_3 relative to M_1, before and after the close encounter, respectively. The maximum distance of approximation between M_2 and M_3 during the encounter is called r_p. The angles involved in the maneuver are: the angle of curvature δ, and the angle between the periapse line and the $M_1 - M_2$ line, called Ψ.

The method of the "patched conics" has the following steps for a complete swing-by:

- At the first stage, the gravitational effect of M_2 is neglected, and so, M_3 is in a keplerian orbit around M_1.
- At the second stage, M_3 crosses the sphere of influence of M_2, with the relative velocity: $\vec{V}_\infty^- = \vec{V}_i - \vec{V}_2$.
- At the third stage, M_3 leaves the sphere of influence of M_2, with the relative velocity: $\vec{V}_\infty^+ = \vec{V}_0 - \vec{V}_2$. At this stage, M_3 reaches a new orbit around M_1, and so, it is said that the swing-by is completed.

The orbit that M_3 reaches after the swing-by presents some differences relative to the previous orbit (before the encounter). The first significant change is the variation of velocity $\Delta\vec{V} = \vec{V}_0 - \vec{V}_i$, that is the difference between the M_3 velocity before and after the encounter. From a diagram of velocity vectors (see Fig. 3) it is possible to demonstrate that [22]:

$$\Delta V = |\Delta\vec{V}| = 2|\vec{V}_\infty| \sin(\delta) = 2V_\infty \sin(\delta) \qquad (1)$$

and that the angle between $\Delta\vec{V}$ and the $M_1 - M_2$ line is $\Psi + 180°$.

The second significant change due to the swing-by is the variation of angular momentum C, given by [22]:

$$\omega\Delta C = -2V_2V_\infty \sin(\delta)\sin(\Psi) \qquad (2)$$

The third significant change is the variation of energy, given by [22]:

$$\Delta E = -2V_2V_\infty \sin(\delta)\sin(\Psi) \qquad (3)$$

In equation 3, V_2 and V_∞ are positive values, since they represent magnitude of vectors. The $\sin(\delta)$ is also always positive, since $0° < \delta < 90°$. Thus, the only parameter that changed the signal of ΔE is the $\sin(\Psi)$, in such way that:

- For $0° < \Psi < 180°$, the swing-by occurs in front of M_2 and the energy of M_3 decreases. The maximum decrease occurs when $\Psi = 90°$.
- For $180° < \Psi < 360°$, the swing-by occurs behind M_2 and the energy of M_3 increases. The maximum increase occurs when $\Psi = 270°$.

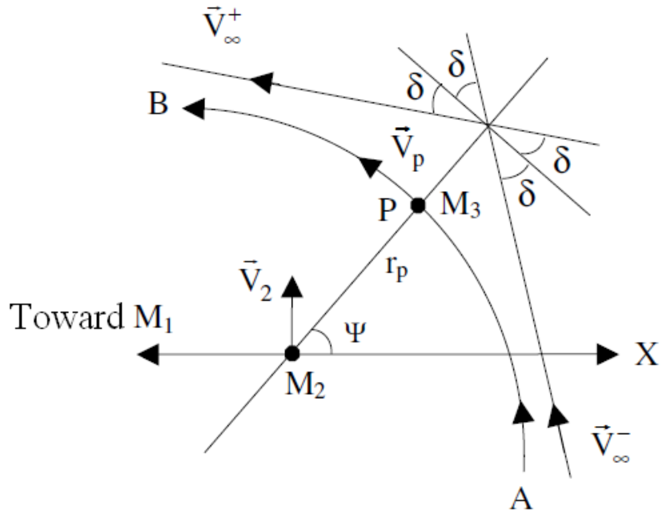

Figure 2: Geometric description and the variables involved in the swing-by [21].

In order to determine the parameters cited above and analyze the change in a given orbit due to the swing-by, its is necessary to determine the velocities and angles involved in the maneuver. This can be done as follow:

1. Calculation of the magnitude of the velocity of M_3 relative to M_1 (V_i).

   When M_3 crosses the sphere of influence of M_2, the distance $M_1 - M_3$ is approximately the distance $M_1 - M_2$. Thus, neglecting the gravitational effect of the planet, we have:

$$|\vec{V}_i| = \sqrt{\mu_1 \left(\frac{2}{r_{12}} - \frac{1}{a} \right)} \tag{4}$$

   where r_{12} is the distance $M_1 - M_2$, considered constant (circular orbit).

2. Calculation of the true anomaly θ at the moment of the encounter.

   At the moment of the encounter $r_{12} = \frac{a(1-e^2)}{1+e\cos(\theta)}$, thus:

$$\cos(\theta) = \frac{1}{e} \left(\frac{a(1-e^2)}{r_{12}} - 1 \right) \tag{5}$$

3. Calculation of the angle (γ) between $\vec{V}_i$ and $\vec{V}_2$ (see Fig. 4).

   The orbit of M_2 was considered circular, thus, (γ) corresponds to the "flight path angle", given by:

$$\tan(\gamma) = \frac{e\sin(\theta)}{1+e\cos(\theta)} \tag{6}$$

   This angle can take on two lagged values of $180°$, corresponding to clockwise and counter-clockwise orbits.

4. Calculation of the magnitude of the relative velocity $\vec{V}_\infty^-$ (see Fig. 4).

   Knowing that the angle between $\vec{V}_2$ and $\vec{V}_i$ is γ, we can apply the law of cosines to found:

$$|\vec{V}_\infty^-| = \sqrt{V_i^2 + V_2^2 - 2V_iV_2\cos(\gamma)} \tag{7}$$

5. Calculation of the angle β between $\vec{V}_\infty^-$ and $\vec{V}_2$ (see Fig. 4).

   Applying again the law of cosines, we found that:

$$\cos(\beta) = -\frac{V_i^2 - V_2^2 - V_\infty^{-2}}{2V_\infty^- V_2} \tag{8}$$

6. Calculation of the deflection angle (δ).

   The angle δ represents a half of the rotation of the velocity vector due to the swing-by. It is given by (Broucke, 1988):

$$\sin(\delta) : \frac{1}{1 + \frac{r_p V_\infty^2}{\mu_2}} \tag{9}$$

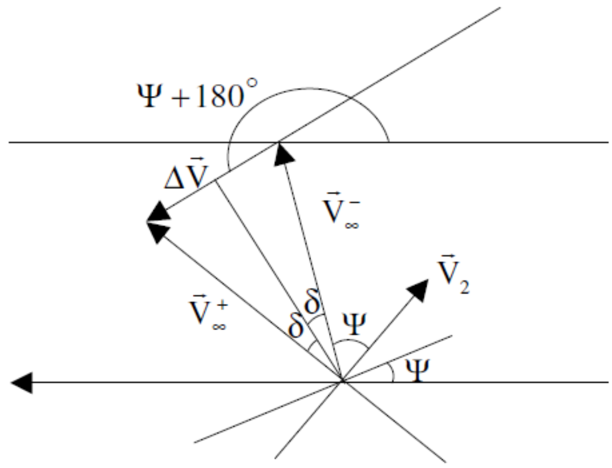

Figure 3: Velocity vectors involved in the swing-by (Prado,2001).

7. Calculation of the approximation angle (Ψ).

   From the geometry of the problem, the possible values for the approximation angle are: $\Psi_1 = 180^\circ + \beta + \delta$ and $\Psi_2 = 360^\circ + \beta - \delta$.

8. Obtaining the variations of energy, angular momentum and velocity.

   The variations of energy ΔE, angular momentum ΔC, and velocity ΔV due to the swing-by are calculated through the previously mentioned equations:

$$\Delta E = -2V_2V_\infty \sin(\delta)\sin(\Psi) \tag{10}$$

$$\omega\Delta C = -2V_2V_\infty \sin(\delta)\sin(\Psi) \tag{11}$$

$$\Delta V = 2V_\infty \sin(\delta) \tag{12}$$

3 Methodology.

The method adopted in this study consists in determine the variation of energy required to change Magnya from its supposed orbit close to Vesta and analyze if such ΔE could be provided by close encounters between Magnya and Vesta.

For the calculation of the initial and final energy, and so, the calculation of variation of energy, we consider the relation:

$$E = -\frac{\mu_1}{2a} \tag{13}$$

where $\mu_1 = GM_1$, being G the universal gravitational constant, and M_1 is the mass of the central-body. In the Vesta-Magnya case, the central-body is the Sun.

For the initial energy we consider that Magnya has orbital elements very close to Vesta, and so, the semi-major axis of Vesta is considered in equation 13, as discussed in section 4.1. For the final energy, we consider the current semi-major axis of Magnya.

Knowing the variation of energy required, we applied the equations that describe the swing-by (Eq. (4) to (12)), for a set of initial conditions of Magnya taken in the vicinity of Vesta, to estimate the ΔE provided by a single, or by multiple close encounters between Vesta and Magnya. Then, we compare these results with the ΔE previously determined, analyzing the role of close encounters in the Vesta-Magnya case.

In the next section we present the results obtained.

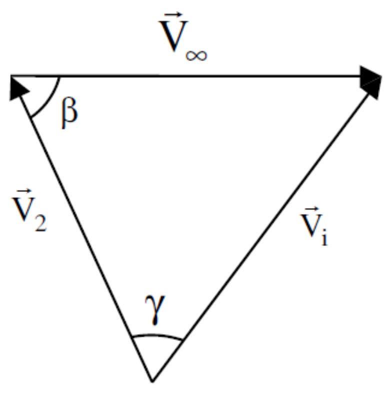

Figure 4: Vector sum involved in the swing-by.

4 Results.

4.1 The single encounter case.

Here we analyze the effects of a single encounter between Vesta and Magnya. We first estimate the variation of energy required to change the orbit of Magnya. After, we apply the swing-by equations for a set of initial conditions, considering a single encounter between the asteroids. We discuss the possibility of the supposed change of orbit be result of the close encounter. Following is the result of such procedure.

1. **Calculation of the energy before the supposed close encounter ($E_{initial}$).**

   Before the supposed encounter the asteroid Magnya should be at the vicinity of Vesta. Thus, in a first approximation, we consider that Magnya has the same semi-major axis of Vesta. Considering such value (see Table 1) in the Eq. (13) we found that the Magnya's initial energy per unit mass is:

   $$E_{initial} = -188.3\ (km^2/s^2)$$

2. **Calculation of the energy after the supposed close encounter (E_{final}).**

   In our approach we consider that a close encounter of Magnya with Vesta has changed its orbit. Thus, we consider that after the close encounter, Magnya has reached a final orbit with semi-major axis equal to 3.14 AU, or 4.69×10^8 km (see Table 1). Considering such value in the Eq. (13), we found:

   $$E_{final} = -141.8\ (km^2/s^2)$$

3. **Calculation of the variation of energy ΔE.**

   Knowing the initial and final values of energy, we calculate the variation of energy required to change the orbit of a body close to Vesta, to an orbit close to Magnya. Being $\Delta E = E_{final} - E_{initial}$, then, we found that:

   $$\Delta E = 46.5\ (km^2/s^2)$$

4. **Calculation of variation of energy due to the swing-by.**

   We simulated one close encounter of Magnya with Vesta, through the equations that describe the swing-by (Eq. (4) to (12)), for a set of initial conditions (semi-major axis and eccentricity) of Magnya in the vicinity of Vesta. The semi-major axis was taken in the range $2.1 \leq a \leq 3.14$ AU, every 0.01 AU. The eccentricity was taken in the range $0.0 \leq e \leq 0.20$.

   The orbit of Vesta around the Sun were considered circular, with the velocity $V_2 = 18.1$ km/s.

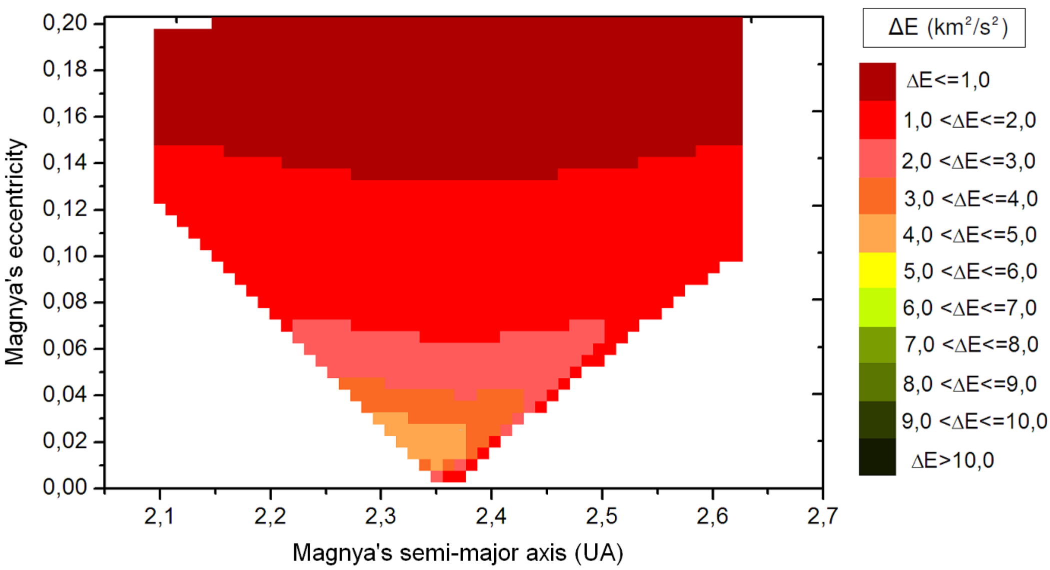

Figure 5: Diagram of variation of energy per unit mass of Magnya due to a close encounter with Vesta for a set of initial conditions of the encounter. The scale on the right represents the gradual change of the energy in (km^2/s^2). The white region represents the set of initial conditions for what the close encounters are not possible, i.e, there is no crossing orbits. For a better visualization we limited the results on diagram to $a = 2.7$ AU.

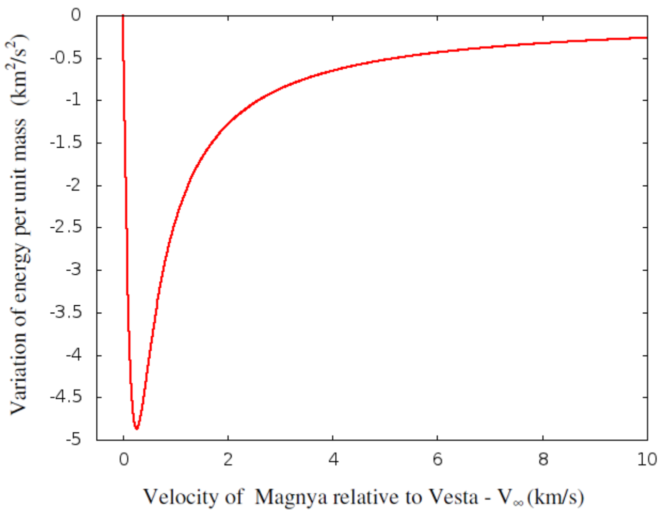

Figure 6: Variation of energy as a function of the relative velocity.

We choose a fixed value for the distance of encounter $r_p = 275$ km, corresponding to ≈ 1.1 radius of Vesta. Taking into account that the radius of Magnya is 15 km, we see then that the surface of the bodies are separated by 10 km during the close encounter, which we considered close enough for our study.

The result of the simulation is presented in a diagram as in Fig. 5. On it we have the variation of energy for each pair $(a \times e)$ of Magnya due to the swing-by. We see from such diagram that the best combinations of initial condition of Magnya is that very close to Vesta (remembering that $a_{Vesta} = 2.36$ AU), and that them lead to a variation of energy with maximum value $\Delta E \approx 5.0\ km^2/s^2$.

Similar result can be found analyzing the variation of the energy due to the swing-by as a function of the relative velocity of the encounter V_∞. This can be done combining the equations 9 and 10, giving:

$$\Delta E = -2V_2V_\infty \frac{\mu_2}{\mu_2 + r_pV_\infty^2} \sin(\Psi) \qquad (14)$$

being $V_2 = 18.1$ km/s (considering the circular orbit of Vesta), and considering the extreme case: $r_p = 1$ radius of Vesta, and $\Psi = 90°$, and therefore, $\sin(\Psi) = 1$. The relative velocity V_∞ was taking going from $V_\infty \approx 0.0\ km/s$ until $V_\infty = 10.0\ km/s$.

The graphic in Fig. 6 shows the variation of energy as a function of such velocity, according to Eq. 14. As demonstrated in the previous diagram, again it is verified that the best combinations of initial conditions lead to a variation of energy with a maximum value $\Delta E \approx 5.0\ (km^2/s^2)$. The best results are found for lower velocities. As the velocity increases, the variation of energy decreases, as expected.

5. Calculation of variation of the semi-major axis and eccentricity.

According to Eq. (13) the energy is related to the semi-major axis. Thus, it is possible, based on the diagram of energy, to determine the diagram of variation of semi-major axis of Magnya after the swing-by with Vesta. Being $E_{after} = E_{initial} + \Delta E$, then:

$$a_{after} = -\frac{\mu_1}{2E_{after}}$$

and:

$$\Delta a = a_{after} - a_{initial}$$

The diagram of Δa for Magnya due to the swing-by with Vesta is presented on Fig. 7. It is clear that the maximum variations of semi-major axis occur for the initial conditions that resulted in the major variation of energy, corresponding to semi-major axis values closer to the semi-major axis of Vesta. However, we verified that the maximum variation of semi-major axis is about $0.06 \leq \Delta a \leq 0.07$ AU, while the variation required would be approximately 0.78 AU (considering the change of semi-major axis of Magnya from 2.36 AU to 3.14 AU (Table 1)).

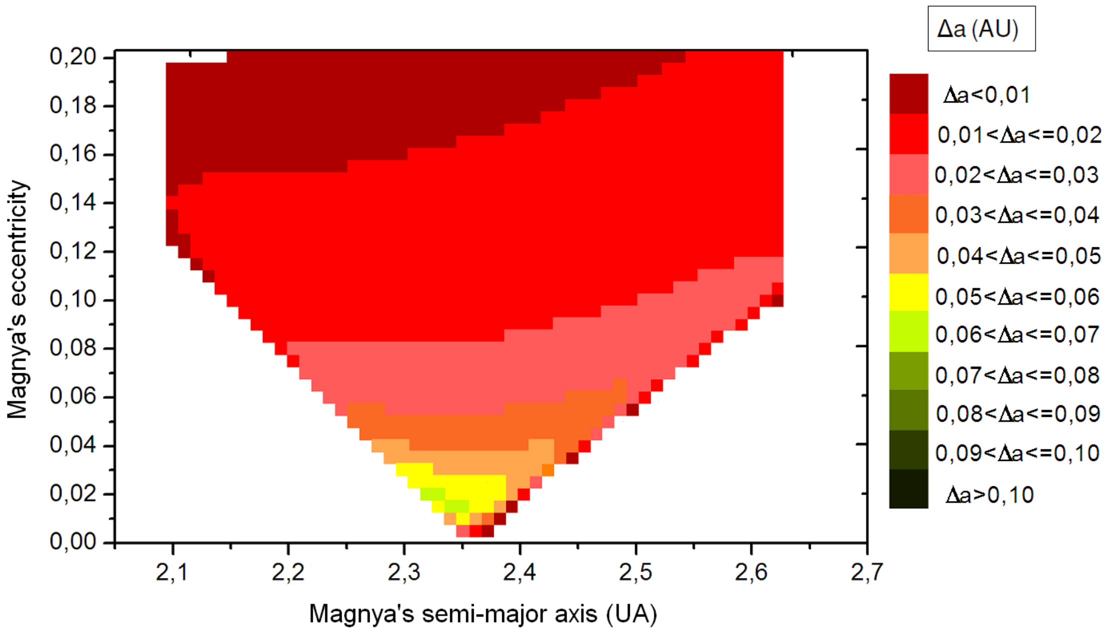

Figure 7: Diagram of variation of semi-major axis of Magnya due to a close encounter with Vesta for a set of initial conditions of the encounter. The scale on the right represents the gradual change of semi-major axis in AU. The white region represents the set of initial conditions for what the close encounters are not possible, i.e, there is no crossing orbits. For a better visualization we limited the results on diagram to $a = 2.7$ AU.

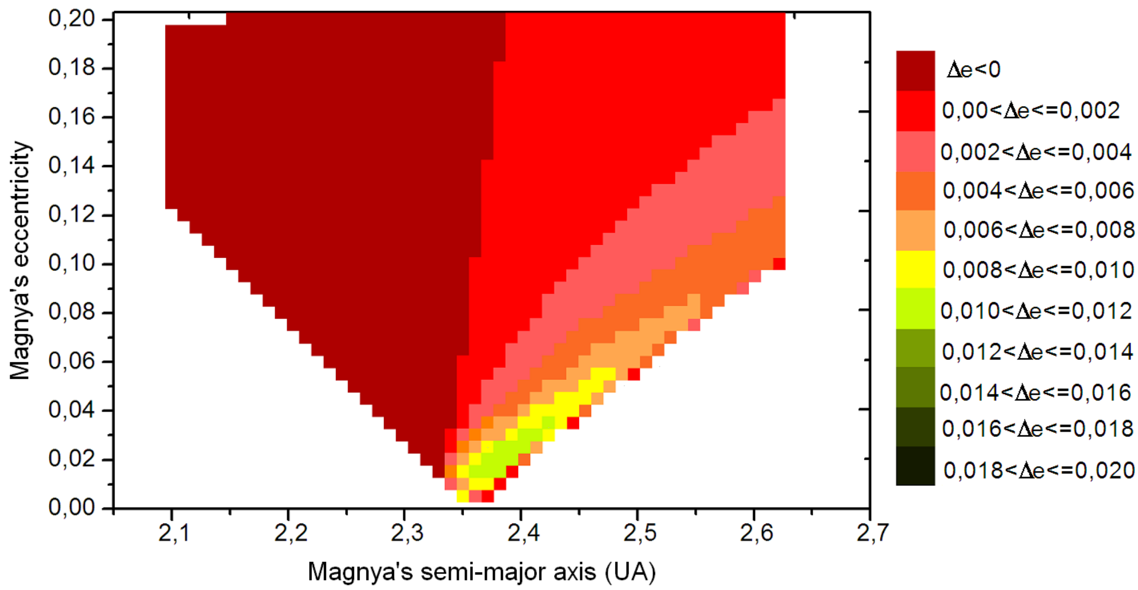

Figure 8: Diagram of variation of eccentricity of Magnya due to a close encounter with Vesta for a set of initial conditions of the encounter. The scale on the right represents the gradual change of eccentricity. The white region represents the set of initial conditions for what the close encounters are not possible, i.e, there is no crossing orbits. For a better visualization we limited the results on diagram to $a = 2.7$ AU.

It is also possible to analyze the variation of eccentricity of Magnya after the swing-by with Vesta, for a set of initial conditions for the encounter. It is known that the eccentricity of an orbit is related to the angular momentum C, through:

$$C = \sqrt{\mu_1 \, a(1-e^2)} \tag{15}$$

being $C_{after} = C_{initial} + \Delta C$, then:

$$e_{after} = \sqrt{1 - \frac{C_{after}^2}{a\mu_1}}$$

and:

$$\Delta e = e_{after} - e_{initial}$$

The diagram of Δe for Magnya due to the swing-by with Vesta is presented on Fig. 8. Again it is clear that the maximum variations of eccentricity occur in the vicinity of Vesta, for the semi-major axis values closer to the semi-major axis of Vesta. However, the maximum variation is about $0.010 \leq \Delta e \leq 0.012$, while the variation required would be approximately 0.14 (considering the change of eccentricity of Magnya from 0.09 to 0.23 AU (Table 1)).

4.2 The multiple encounters case.

Now that we have analyzed the single encounter case, we are going to discuss the effects in the orbit of Magnya due to possible multiple encounters with Vesta.

Through the diagrams of variation of energy, semi-major axis and eccentricity, presented in Fig. 5, 7 and 8, it is possible to follow a sequence of close encounters between both asteroids. This is done considering the initial and final orbits of the asteroids.

The initial orbit corresponds to a given orbit before a supposed encounter with the semi-major axis a and the eccentricity e. After the encounter, the asteroid reaches a new orbit with $a_{after} = a + \Delta a$ and the eccentricity $e_{after} = e + \Delta e$. Thus, we consider the parameters of the new orbit as the initial condition for the next close encounter.

We exemplify this procedure for the Vesta-Magnya problem with an arbitrary case. Considering a close encounter of Magnya with Vesta, with the initial conditions: $a = 2.37$ AU, and $e = 0.025$. It is known, by the diagram of ΔE, Δa and Δe, that in such condition the asteroid Magnya will suffer a variation of $4.0 \leq \Delta E \leq 5.0 \ (km^2/s^2)$, $0.05 \leq \Delta a \leq 0.06 \ (AU)$ and $0.008 \leq \Delta e \leq 0.010$. However, through simulation it is known that the exact values are: $\Delta E = 4.00 \ (km^2/s^2)$, $\Delta a = 0.052 \ (AU)$ and $\Delta e = 0.008$, in such way that the new orbit of Magnya has the semi-major axis equal to $a + \Delta a = 2.42$ AU and the eccentricity $e + \Delta e = 0.033$. Thus, these new values are considered as the initial conditions for Magnya for the next close encounter. The complete sequence for the arbitrary example considered here is presented in Tab. 2 and is demarcated by the white filled circles in the diagram of the variation of energy reproduced in Fig. 9.

For the example considered here, we found that it is possible for Magnya to perform three close encounter with Vesta, resulting in a $\Delta E \approx 9 \ km^2/s^2$. After those encounters, we see that Magnya reaches a region where the close encounters with Vesta are no longer possible. From this example it is clear that although the multiple encounters between the asteroids are possible there is a limit, given by the possibility of close encounters to happen. As the semi-major axis and the eccentricity of Magnya increase the probability of crossing the orbit of Vesta decreases, until values that the encounters are no longer possible. From the previous diagrams (Figs. 5, 7 and 8), we see that the upper limit for semi-major axis, that allows the encounters to happen, is ≈ 2.6 AU, and that at this distance the possible ΔE that Magnya may suffer is relatively small. Therefore, even considering multiple swing-bys, we conclude that the supposed change of orbit of Magnya was not cause only by close encounters.

5 Final comments.

In the present work we considered the effects of close encounter between Vesta and Magnya in order to explain the orbital location of Magnya. The equations that describe the swing-by were applied.

We have shown that energy variations achieved through a single encounter are very small when compared to the variation that would be required to change the orbit of Magnya. The best combinations of initial condition of Magnya lead to a variation of energy with maximum value $\Delta E \approx 5.0 \ km^2/s^2$ while the required would be $\approx 47 \ km^2/s^2$. As consequence of such small variation of energy, the variation of semi-major axis and eccentricity are not also enough to explain the distant orbital position of Magnya relating to the asteroid of the Vesta family.

Table 2: Sequence of multiple encounters between Vesta and Magnya.

	Before the encounter		After the encounter		
Encounter	$a_{Magnya}(AU)$	e_{Magnya}	$a_{Magnya}(AU)$	e_{Magnya}	$\Delta E(km^2/s^2)$
1	2.37	0.025	2.42	0.033	4.00
2	2.42	0.033	2.47	0.044	3.09
3	2.47	0.044	2.49	0.05	1.63
4	2.49 *	0.05 *	-	-	Total = 8.72

* no more encounters, i.e., there is no more crossing orbits for such initial condition.

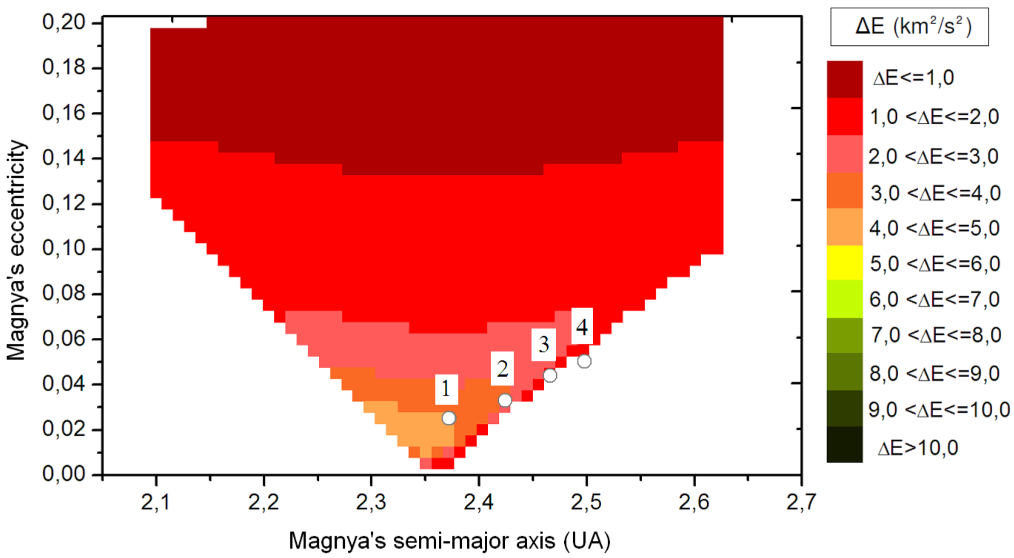

Figure 9: Reproduction of the diagram of the variation of energy of Magnya due to a close encounter with Vesta for a set of initial conditions of the encounter. The white circles mark a sequence of close encounters between Magnya and Vesta for an arbitrary initial condition. The white region represents the set of initial conditions for what the close encounters are not possible, i.e, there is no crossing orbits.

The effects of multiple encounters were also considered. We found that there is a limit for the close encounters to happen, i.e., as the semi-major axis and eccentricity of Magnya increase due to the swing-by, it reaches a region where the close encounters with Vesta are no longer possible.

Such results indicates that the supposed change of orbit suffered by Magnya was not caused by close encounters, i.e., the change of orbit was not caused only by the swing-by, and that therefore, the asteroid Magnya would have to suffer other perturbations (such as resonances, collisions or close encounters with other bodies, for instance) that would provide such change on its orbit.

6 acknowledgment

The authors acknowledge the financial support from INCT - Estudos do Espaço,CNPq and FAPESP.

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