

# Exploiting outage performance of imperfect hardware at relay in Cooperative NOMA Networks

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*Abstract:* - Thanks to improvement in successive interference cancellation (SIC), non-orthogonal multiple access (NOMA) can be implemented in 5G communications. In this paper, the cooperative NOMA (C-NOMA) systems is investigated in two proposed schemes, namely perfect hardware cancellation (PC) and imperfect hardware cancellation (IC) considering on efficiency of hardware noise cancellation operations. The first scenario is that perfect processing between the relay and the far user. In the second scenario, the hardware noise resulted by imperfect circuit at relay for NOMA operation. To illustrate the performance of C-NOMA in two considered scenarios, the closed-form expressions for both exact and asymptotic outage probability are derived for each NOMA user. Simulation results validate that the outage performance of C-NOMA with IC scheme is superior to C-NOMA with PC scheme at low SNR region rather than at high SNR region.

*Key-Words:* - NOMA, outage probability, hardware noise cancellation

## 1 Introduction

Wireless communications with key technologies related to radio access was investigated [1]. As potential candidate access technology, non-orthogonal multiple access (NOMA) is introduced to adapt to requirements of high traffic volume and spectral consuming efficiency, and such paradigm is recommended for the fifth generation (5G) [2]-[3]. By splitting signal power in the power domain at the transmitter, NOMA superposes the signals intending to serve multiple users. In other side, successive interference cancellation (SIC) can be applied in the receiver and such technique is exploited to distinct multiplexed users' signals [4]. As a key feature of NOMA, user fairness is designed to satisfy spectrum utilization. In principle, NOMA assigns less power to users who obtain better channel and more power to users who undergo worse channel conditions. Such unbalanced power allocation is required to realize an enhanced trade-off between user fairness with system throughput. It can be considered NOMA has power allocation which is dissimilar with conventional water-filling power allocation. In NOMA, multiple users are assisted to access to network at the same time, frequency and spreading

codes but different power level which result in an escalation in user fairness and spectral efficiency.

More specifically, the ergodic capacity maximization problem can be examined in multiple-input-multiple-output (MIMO) NOMA networks [5] and they developed an optimal power allocation scheme. In cellular downlink NOMA networks as in [6], the outage behaviour and the ergodic sum rate was investigated. Recent years, cooperative NOMA (C-NOMA) networks have involved in large number of publications. In order to enhance the system capacity and the reliability of NOMA, the authors in [7] considered the achievable average rate for C-NOMA networks. In [8], multiple-antenna C-NOMA network was developed to improve the system performance, and the authors also investigated system outage performance. To improve the spectral efficiency, the authors in [9] shown the directed and relay transmission as two methods applied in NOMA. In [10], a suboptimal and practical power allocation strategy are evaluated in novel detection scheme for the C-NOMA network.

To guarantee the quality of service, as in [11], the outage probability and ergodic sum rate are measured in case of random placement of users and they examined the circumstance in a downlink NOMA system. In addition, the sum rate of users can be optimized in an uplink NOMA system as in [12]. In scenario of fixed-power allocation NOMA system, the authors in [13] calculated the influence of user coupling on the sum rate and such model can be extended to evaluate a cognitive radio assisted NOMA system. In addition, efficient power distribution schemes are studied in both downlink and uplink NOMA systems as investigation in [14]. In term of maximizing the minimum achievable rate by determining instantaneous/average channel state information at the transmitter and corresponding power allocation schemes are proposed to apply in the NOMA system [15]. Besides, outage probability and sum rate performance are investigated in several works such as [16], [17], in which relaying schemes are adopted to support the transmission between two transceivers.

So far, ideal SIC conditions is deployed in most of the research work shown on SIC operation in considered NOMA. More specifically, the NU receiver perfectly cancel interference from the FUs, and this scenario corresponds with the situation as the NU has perfect information of the FUs signal. Unfortunately, very limited work has been considered as the non-ideal case where imperfect SIC condition is considered. Under impacts of distorted effects SIC processing at the NU on Rayleigh fading channel, such imperfect signal appears to be a more realistic scenario in NOMA [18]. In addition, imperfect hardware affects on relaying system performance as recent results in [19]. This motivate us to provide the performance of imperfect hardware in NOMA, hence introducing a more realistic analysis as compared to the idea conditions of hardware at relay. Particularly, two NOMA relay schemes related SIC efficiency are investigated and evaluated. In C-NOMA system model, the signal processing experiences in two successive time slots to help NOMA information transfer. In the first slot the base station (BS) uses NOMA scheme to conduct the superimposed signal to relay which equips SIC circuit. Upon receiving the signal, the considered relay will decode the signals by using perfect or imperfect hardware and then sending the preceded signals to the intended recipients in the next time slot.

## 2 Imperfect hardware scheme in NOMA

Considering on downlink wireless communication system that contains of one base station (BS) and representative users (UEs). Each UE can be considered as relay or destination where are located at far distance or near distance. We denote  $h, g$  are channel in the first hop and the second hop in relaying communications in which these channel are the Rayleigh fading channel gain. The transmit powers of BS and UE are  $P_s, P_r$ . Note that the focus of this paper is to study the impact of SIC efficiency on the performance of NOMA, which will provide important insights for the design of effective transmission scheme in NOMA.

In the literature, the system performance for user 1 and user 2 in NOMA is usually investigated by assuming the relay can decode NOMA signals exactly by employing SIC. In this scenario, regarding on the concept of error of hardware noise, imperfect hardware processing can affect the system performance related to outage probability. In particular, the imperfect cancellation (IC) related to hardware in NOMA results in hardware noise (HN) term in received signal. As a result, the near user (NU) obtains the superposed signal and hardware noise (HN) signal simultaneously. In this case, the observation at  $NU$  can be expressed by:

$$y_{NU}[k] = h(\sqrt{\alpha P_s} x_1[k] + \sqrt{(1-\alpha) P_s} x_2[k]) + v[k] + n_{NU}[k] \quad (1)$$

where  $v$  denotes distortion noise from hardware noise due to imperfect hardware at the relay node  $NU$ , and  $v \sim CN(0, \kappa^2 P_s |h|^2)$  [19]. We denote the power allocation coefficient for  $x_1$  and  $x_2$  signal as  $\alpha$  and  $1-\alpha$ , respectively and  $n_{NU}$  denotes as Gaussian noise term with variance of  $N_0$ . It is assumed that  $\alpha < 0.5$  as the basic fundamental of NOMA assigned more power for the far users and less power for the near users. It worth noting that successive interference cancellation (SIC) in NOMA is deployed at  $NU$ . Therefore, the received signal to noise ratio (SNR) at  $NU$  to detect the  $FU$ 's message is formulated by

$$\gamma_{FU,NU} = \frac{(1-\alpha)\rho|h|^2}{|h|^2\alpha\rho + \kappa^2|h|^2\rho + 1} \quad (2)$$

where  $\rho = \frac{P_s}{N_0}$  stands for signal-to-noise ratio

(SNR). It can be assumed that  $E\{x_1^2\} = E\{x_2^2\} = 1$

After performing SIC, due to imperfect SIC the received SNR at *NU* to detect its own message  $x_1$  is given by

$$\gamma_{NU} = \frac{\alpha\rho|h|^2}{|\kappa^2|h_{HN}|^2\rho + 1} \quad (3)$$

In this NOMA system, the superposed signals of expected users at *NU* is decoded and forwarded to *FU*. The observation at the far user (*FU*) can be expressed as

$$y_{FU}[k-\tau] = g\sqrt{\eta P_r}x_2[k-\tau] + n_{FU}[k-\tau] \quad (4)$$

In which  $\eta$  denotes as power percentage for signal forwarding processing and remain percentage for SIC operation.

Therefore, the received SNR is given by

$$\gamma_{FU} = |g|^2\eta\rho \quad (5)$$

## 2.1 Outage Probability of NU

In NOMA related to imperfect hardware (IC case), the outage probability is considered as an important metric for performance evaluation. Following the pre-defined target rate of users, quality of service (QoS) can be verified for each applications applied in wireless communications. In this subsection, the first scenario is investigated in terms of outage probability.

According to the NOMA protocol, the complementary events of outage at *NU* can be explained as below equation. Firstly, we explore the outage probability of *NU* and it can be shown as:

$$OP_{NU} = 1 - \Pr(\gamma_{FU,NU} > \gamma_{02}, \gamma_{NU} > \gamma_{01}) \quad (6)$$

where  $\gamma_{01} = 2^{R_1} - 1$  is the threshold SNR with  $R_1$  is denoted as the target rate at *NU* to detect  $x_1$  and  $\gamma_{02} = 2^{R_2} - 1$  with  $R_2$  is denoted as the target rate at *FU* to detect  $x_2$

The closed-form expression for the outage probability of *NU* is given by

$$OP_{NU} = 1 - \Pr(A > \gamma_{02}, b > \gamma_{01}) \quad (7)$$

where  $A = \frac{(1-\alpha)\rho|h|^2}{|h|^2\alpha\rho + \kappa^2|h_{HN}|^2\rho + 1}$ ,  
 $B = \frac{\alpha\rho|h|^2}{|\kappa^2|h_{HN}|^2\rho + 1}$ . Then, it can be re-written as

$$OP_{NU} = 1 - \Pr(\rho|h|^2[(1-\alpha) - \alpha\gamma_{02}] > \gamma_{02}A, \alpha\rho|h|^2 > \gamma_{01}A),$$

It is noted that we denote  $A = \kappa^2|h_{HN}|^2\rho + 1$ , it can be obtained as

$$OP_{NU} = 1 - \Pr(|h|^2 > \frac{\gamma_{02}A}{\rho[(1-\alpha) - \alpha\gamma_{02}]}, |h|^2 > \frac{\gamma_{01}A}{\alpha\rho}) \quad (9)$$

$$\text{We set } a_1 = \frac{\gamma_{01}}{\alpha\rho}, a_2 = \frac{\gamma_{02}}{\rho[(1-\alpha) - \alpha\gamma_{02}]}$$

In this paper we assume that  $a_i > a_j, i, j = 1, 2$ . Applying the probability density function (PDF) for

$$\text{Rayleigh channel } f_{|h_i|^2}(x) = \frac{1}{\Omega_{|h_i|^2}} \exp\left(-\frac{x}{\Omega_{|h_i|^2}}\right), x \geq 0$$

, we obtain the following outage probability

$$OP_{NU} = 1 - \frac{\Omega_h}{\Omega_h + \rho\Omega_h\kappa^2a_i} e^{-\frac{a_i}{\Omega_h}} \quad (10)$$

Considering on the special case of perfect SIC at NOMA, the outage probability can be rewritten as

$$OP_{NU} = 1 - e^{-\frac{a_1}{\Omega_h}} \quad (11)$$

## 2.2 Outage Probability of FU

The outage events of  $FU$  can be seen for two main reasons. The first reason is that  $NU$  can not detach  $x_2$  and the second reason is that  $FU$  cannot detect its own message on the good conditions that  $NU$  still detach  $x_2$  effectively. As a result, the outage probability of  $FU$  can be expressed based on such analysis

$$OP_{FU} = \Pr(\gamma_{FU,NU} < \gamma_{02}) + \Pr(\gamma_{FU,NU} > \gamma_{02}, \gamma_{FU} < \gamma_{02}) \quad (12)$$

To proceed the closed-form expression of outage probability, it can be re-written as

$$OP_1 = \Pr(\gamma_{FU,NU} < \gamma_{02}) = \Pr\left(\frac{(1-\alpha)\rho|h|^2}{|h|^2\alpha\rho + \kappa^2|h_{HN}|^2\rho + 1} < \gamma_{02}\right) \quad (13)$$

Similarly, such outage probability can be computed as

$$OP_1 = 1 - \frac{\Omega_h}{\Omega_h + \rho\Omega_h\kappa^2a_2} e^{-\frac{a_2}{\Omega_h}} \quad (14)$$

We continue compute the second term as below

$$OP_2 = \Pr(\gamma_{FU,NU} > \gamma_{02}, \gamma_{FU} < \gamma_{02}) = \Pr\left(\frac{(1-\alpha)\rho|h|^2}{|h|^2\alpha\rho + \kappa^2|h_{HN}|^2\rho + 1} > \gamma_{02}, |g|^2\eta\rho < \gamma_{02}\right) \quad (15)$$

Similarly, such outage probability can be expressed by

$$OP_2 = 1 - \frac{\Omega_h}{\Omega_h + \rho\Omega_h\kappa^2a_2} e^{-\frac{a_2}{\Omega_h}} \left(1 - e^{-\frac{\gamma_{02}}{\Omega_g\eta\rho}}\right) \quad (16)$$

Finally, the outage probability of  $FU$  can be written as

$$OP_{FU} = 1 - \frac{\Omega_h}{\Omega_h + \rho\Omega_h\kappa^2a_2} e^{-\left(\frac{a_2}{\Omega_h} + \frac{\gamma_{02}}{\Omega_g\eta\rho}\right)} \quad (17)$$

## 3 Perfect hardware scheme in NOMA

Regarding on perfect hardware noise cancellation (PC case) at relay in NOMA. In particular, the received signal at  $NU$  obtains the superposed signal. The observation at  $NU$  can be expressed by:

$$y_{NU}[k] = h(\sqrt{\alpha P_s}x_1[k] + \sqrt{(1-\alpha)P_s}x_2[k]) + n_{NU}[k] \quad (18)$$

In this scenario, the outage performance of the  $NU$  can be computed as

$$OP_{NU} = 1 - e^{-\frac{a_1}{\Omega_h}} \quad (19)$$

Next, the outage performance of the  $NU$  st perfect case of SIC can be expressed as

$$OP_{FU} = 1 - e^{-\left(\frac{a_2}{\Omega_h} + \frac{\gamma_{02}}{\Omega_g\eta\rho}\right)} \quad (20)$$

**Remark 1:** At high SNR which equivalent with  $\rho \rightarrow \infty$  then it can be applied  $e^z \approx 1 - z$ ,  $z \rightarrow 0$  the asymptotic outage probability of  $NU$  for C-NOMA is given by

$$OP_{asym,NU} = \frac{\max\{a_i\}}{\Omega_h} \quad (21)$$

Similarly, the asymptotic outage probability of  $FU$  for C-NOMA is expressed by

$$OP_{asym,FU} = \frac{a_2}{\Omega_h} + \frac{\gamma_{02}}{\Omega_g\eta\rho} \quad (22)$$

## 4 Numerical Results

In this section, numerical and analytical simulation results are deployed to corroborate our derived expressions related to performance

evaluation of NU and FU. In these simulations, we assume that the distance between pair of node is normalized to unity  $\Omega_h = \Omega_g = 1$ . For user relaying satisfying expected QoS, the target rate is designed to be  $R_1 = 3, R_2 = 1$  (bps/Hz)  $R_1 = 3$ , for NU and FU, respectively. Without loss of generality, the power allocation fraction of NOMA is  $\alpha = 0.25$ . In NOMA, we assume power allocated for SIC is 10%. It is worth noting that the communication process in period is completed in two slots as discussion in previous section.

Fig. 1 demonstrates the outage probability with pre-defined power allocations for the NU located in NOMA for each user in the C-NOMA system for both IC and PC case. In this experiment, the lines of the theoretical analysis match the line of the simulation results perfectly. Furthermore, it can be seen that the outage performance will be better as the SNR of the BS grows and converges to a certain level when SNR is large enough. Regarding on impacts of hardware noise, this indicates that the outage probability in the C-NOMA system can be reduced with a lower hardware noise due to imperfect noise cancellation technique.

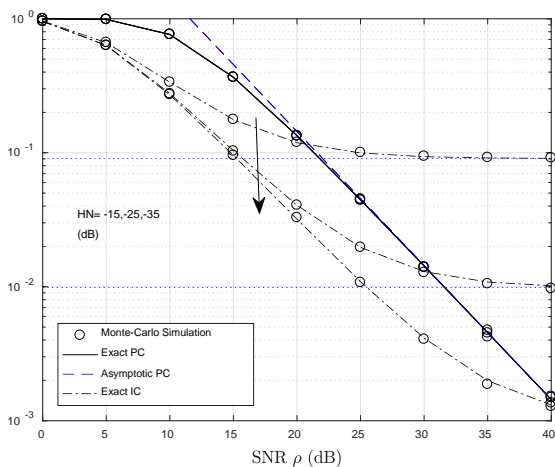


Fig. 1. Outage performance of NU versus transmit SNR at the BS

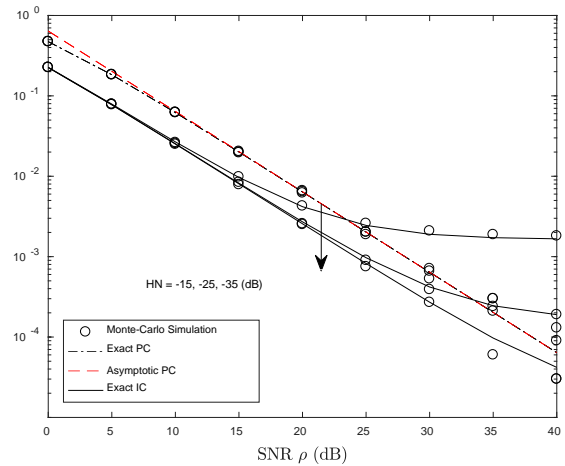


Fig. 2. Outage performance of FU versus transmit SNR at the BS

Fig. 2 demonstrates the outage probability with pre-defined power allocations for the FU located in the C-NOMA system. In this experiment, higher power allocation for this far user can be experienced better outage performance compared low power. It is noted that at high SNR, outage performance remains constant.

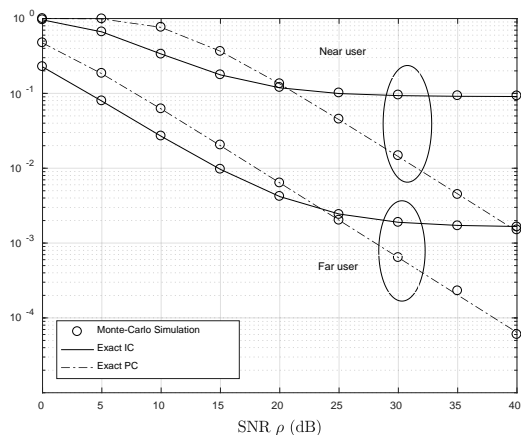


Fig. 3. Comparison study on outage performance of NU and FU

Fig. 3 illustrates the impact of hardware noise in FU and NU regarding on the outage probability. From this figure, for a low given SNR (approximately less than 25 dB), the curve of the outage probability in IC outperforms PC and it is contrast trend in the higher SNR. Meanwhile, we observe that the minimum outage probabilities can be obtained in perfect hardware impairment.

## 5 Conclusion

In this paper, a C-NOMA with SIC capability scheme have been proposed. This novel

C-NOMA outage performance analysis scheme can be applied to evaluate downlink of this C-NOMA scenario. The analytical expressions have been derived for the outage probability and exactness of these expressions are verified in simulation results. The derived analytical expressions show that C-NOMA with PC and IC schemes can achieve the required outage performance under condition of low hardware noise, and careful choice of power allocation leads to better performance. One promising future direction is to combine this novel C-NOMA power allocation scheme with hardware noise cancellation scheme in a relaying network.

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