Calculation of the Flow Characteristics in the Offshore Gas Pipelines in the Northern Seas with Accounting for Growing and Ice Melting on its Outer Surface

KURBATOVA GALINA, ERMOLAEVA NADEZHDA, NIKITCHUK BOGDAN
Electromechanical and Computer Systems Modelling
Saint Petersburg University
Universitetskii prospekt 35, Petergof, Saint Petersburg, Russia 198504
RUSSIA
n.ermolaeva@spbu.ru http://www.apmath.spbu.ru/en/staff/ermolaeva/index.html

Abstract: - In the present work, an effective algorithm for a numerical solution to the equations system modelling turbulent flow of a gas mix under a high pressure through the long offshore gas pipelines is presented. We pay particular attention to the heat exchange problem of the gas flow with the ambient through the multilayer wall of pipeline with accounting for the growing and melting processes of sea ice. Developed model allows taking into account the seasonal changes of sea temperature.
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Key-Words: - Offshore gas pipelines, sea ice, pipeline glaciation, unsteady heat exchange, ice melting.

1 Introduction

Design of offshore gas pipelines, monitoring of operated ones, investigation of safety matters and influence of the ecological situation in the water area require the development of an adequate pipeline gas flow model. The research of the gas flow in pipelines begun in the classical works of C. F. Colebrook, I. Nikuradze, A. J. Reynolds, T. Karman, S. K. Godunov, L. G. Loitsyanskii, I. A. Charnyi, G. Schlichting and many other scientists. This problem is still greatly relevant to date. A notable contribution to modelling gas flow in pipelines has been made by M. Chaczykowski [1], A. J. Osiadacz [2], O. F. Vasilev, Ye. A. Bondarev, A. F. Voyevodin, and M. A. Kanibolotskii [3], J. F. Helgaker [4], V. I. Zubov, V. N. Koterov, V. M. Krivtsov and A. V. Shipilin [5], L. M. C. Gato and J. C. C. Henriques [6]. Despite the amount of literature concerned with this problem, modelling the gas flow through offshore pipelines is far from completion. Northern seas modelling is complicated by the necessity to take into account a possibility of outer surface pipeline glaciation. The present work investigates making effective algorithms for the gas flow characteristics calculated in the offshore gas pipelines, and for possible pipeline glaciations in northern seas.

2 Model Formulation

A quasi-one-dimensional model of the nonstationary nonisothermal flow of a real gas mix through an offshore gas pipeline operating under conditions with the possibility of pipeline glaciation can be written as:

continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial z} = 0, \qquad (1)$$

equation of motion

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial \left(\rho u^2 + p\right)}{\partial z} = -\frac{\lambda \rho u |u|}{4R} + \rho g \cos\tilde{\varphi}, \qquad (2)$$

energy equation

$$\frac{\partial (\rho e)}{\partial t} + \frac{\partial \left(\rho u \left(e + \frac{p}{\rho}\right)\right)}{\partial z} = -\frac{2q}{R} + \rho u g \cos\tilde{\varphi}, \qquad (3)$$

relation between energy and internal energy

$$e = \varepsilon + u^2/2, \qquad (4)$$

the Redlich-Kwong equation of state

$$p = \frac{h\rho T}{1-\rho\delta} - \frac{c\rho^2}{(1+\rho\delta)\sqrt{T}}, \qquad (5)$$

caloric equation of state

$$\varepsilon = \tilde{c}_v T - \frac{3}{2}\frac{c}{\delta\sqrt{T}}\ln(1+\rho\delta), \qquad (6)$$

$$q = q\left(R, \delta_1, \delta_2, T, t, y(t), T^*, \beta\right), \qquad (7)$$

the initial conditions for $\rho,\ u,\ T,\ y,$ (8)

the boundary conditions for $\rho,\ u, T,\ u.$ (9)

In model (1)-(9) the processes in gas flow are described in terms of cross-sectional averaged pressure, in the velocity, in the temperature and etc. We use the following designations: $u,\ \rho,\ p,\ T,\ e,\ \varepsilon$ are the flow velocity, the density, the pressure, the temperature of a gas mixture, the mass densities of energy and internal energy correspondingly, which are functions of time t and coordinate z coinciding with the gas pipeline axis; R is the inner radius of the gas pipeline; g is the gravity acceleration; $\tilde{\varphi} = \tilde{\varphi}(z)$ is the angle between the gravity vector and z-axis. Calculation of heat flux vector q (7) is performed by using the nonstationary heat transfer model given below; $\delta_1,\ \delta_2$ are the thicknesses of the layers comprising the coat of the gas-pipeline; T^* is the ambient temperature; $y(t)$ is the ice thickness in z-th cross-sections; β is the total heat transfer coefficient; λ is the hydraulic resistance coefficient; h, c, δ are constants in the Redlich-Kwong equation of state (5) determined by a given chemical composition of a gas mixture [7]; $\tilde{c}_v$ in equation (6) is the specific heat of an ideal gas (including ideal gas mixtures).

One of the stationary variant of this model was successfully used in simulating the gas flow in the gas pipeline "North European Gas Pipeline" (Nord Stream), running from Portovaya Bay near Vyborg in the Russian Federation to Lubmin in Germany. This model was detailed in the book "Models of sea gas-pipelines" [8].

3 Heat Exchange Model

In the northern seas pipeline glaciation is possible [9]. An ice layer affects the heat transfer processes between the gas flow and the environment as well as the pipeline buoyancy.

We studied [10] the heat transfer processes between the gas and the pipeline surroundings, taking into account the ice layer growing (in northern seas). We developed a selection procedure of the thermophysical characteristics of growing ice in sea water and the Stefan condition modification, which reflects the features of glaciation in sea water. The obtained results were quite coherent with the experimental data. The mathematical model of unsteady heat transfer through the multilayer wall of pipeline, involving the modified Stefan condition, is given by:

$$\rho_1 c_1 \frac{\partial T_1}{\partial t} = \frac{\lambda_1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_1}{\partial r}\right), \quad r \in \left(R, R_1\right), \quad t > t_*, \tag{13}$$

$$t = t_*, \qquad T_1(r) = T_1^0(r), \tag{14}$$

$$t > t_*,\ r = R:\ -\alpha_0\left(T(z,t) - T_1\right) = \lambda_1\frac{\partial T_1}{\partial r} = q, \tag{15}$$

$$t > t_*, \quad r = R_1 : T_1 = T_2,\ \lambda_1\frac{\partial T_1}{\partial r} = \lambda_2\frac{\partial T_2}{\partial r}, \tag{16}$$

$$\rho_2 c_2 \frac{\partial T_2}{\partial t} = \frac{\lambda_2}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_2}{\partial r}\right),\ r \in \left(R_1, R_2\right),\ t > t_*, \tag{17}$$

$$t = t_*, \qquad T_2(r) = T_2^0(r), \tag{18}$$

$$t > t_*,\ \ r = R_2:\ T_2 = T_4,\ \lambda_2\frac{\partial T_2}{\partial r} = \lambda_4\frac{\partial T_4}{\partial r}, \tag{19}$$

$t > t_*, \quad r \in \left(R_2, R_2 + y(t)\right):$

$$\rho_4 c_4 \frac{\partial T_4}{\partial t} = \frac{\lambda_4}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_4}{\partial r}\right), \tag{20}$$

$$t = t_*, \quad y = y_0, \quad T_4(r) = T_4^0(r), \tag{21}$$

$$t > t_*, \quad r = R_2 + y(t):\quad T_4 = T_*, \tag{22}$$

$$t > t_*, \qquad \lambda_4\left.\frac{\partial T_4}{\partial r}\right|_{R_2 + y(t)} - q_3 = \left(\gamma\rho_4 + \alpha\right)\frac{dy}{dt}. \tag{23}$$

Model equations (13), (17), (20) are the heat equations in the pipeline wall layers and in the growing ice layer, respectively; (23) is the modified Stefan condition. Condition (22) expresses the invariableness temperature on a glaciation front. Here r is the radial coordinate in cylindrical coordinate system $\left(r, \varphi, z\right)$; while $\rho_k,\ \lambda_k,\ c_k$ and $T_k = T_k(r,t)$ is the density, the coefficient of thermal conductivity, the specific heat and the temperature distribution in k-th layer; indices $k = \overline{1,4}$ correspond to steel and concrete layers of pipeline wall, the thermal boundary layer of water and the ice layer respectively; γ is the latent heat (of fusion) of ice. α is an effective parameter, accounting in particular angle averaging, as one-dimensional problem supposes the axial-symmetric of processes - besides, the parameter α allows accounting for dissimilarity between glaciation processes in salty and fresh water; α_0 is the heat transfer coefficient between the gas and the inner wall. $T(z,t)$ is the gas temperature. y_0 is the ice thickness at the initial time; q_3 is the radial component of the heat flux vector from seawater to glaciation front with coordinate $r = R + y(t)$; $R_1 = R + \delta_1,\ \ R_2 = R_1 + \delta_2,\ \ T_*$ is the seawater-ice transition temperature; t_* is the time moment at which in this section the ice layer occurs. For glaciation beginning two conditions must hold

$$T_2(R_2,t_*) \le T_*, \tag{24}$$

$$\lambda_2 \frac{\partial T_2}{\partial r}\bigg|_{R_2,t_*} > \lambda_3 \frac{\partial T_3}{\partial r}\bigg|_{R_2,t_*}. \tag{25}$$

For modelling heat transfer for $t < t_*$ we use model (13)-(18), which is supplemented by the heat equation in the thermal boundary layer of water with the thickness δ_*:

$$\rho_3 c_3 \frac{\partial T_3}{\partial t} = \frac{\lambda_3}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_3}{\partial r}\right), \quad r \in \left(R_2, R_2+\delta_*\right), \tag{26}$$

$$t = t_0, \quad T_3(r) = T_3^0(r), \tag{27}$$

$$r = R_2: \ T_2 = T_3, \ \lambda_2 \frac{\partial T_2}{\partial r} = \lambda_3 \frac{\partial T_3}{\partial r}, \tag{28}$$

$$r = R_2 + \delta_*, \quad T_3 = T^*(t). \tag{29}$$

The thickness of the thermal boundary layer of water can be expressed through the total heat transfer coefficient β as follows:

$$\delta_* = R_2\left(\exp\left(\frac{\lambda_3}{\beta R}\left(1-\beta R\left(\frac{1}{\lambda_1}\ln\frac{R_1}{R}+\frac{1}{\lambda_2}\ln\frac{R_2}{R_1}\right)\right)\right)-1\right).$$

The total heat transfer coefficient β and the hydraulic resistance coefficient λ are determined using the identification procedure [11].

From solution to the system (13)-(18), (26)-(29) the values t_*, q_3 are found. The functions in model (13)-(23) parametric depend on z and t via T and $q_3\left(T^*(z,t)\right)$. If the ambient temperature $T^*(z,t)$ increases over time, the formed ice begins to melt. Model (13)-(23) allows taking into account as a grow of an ice layer as well as one's melting. Under melting conditions the value α in (23) is equal to 0.

3.1 Numerical Algorithm

For the numerical solution to the model equations (13)-(23) we use the grid method with explicit separation of the moving phase-change surface, (the front-racking method) [12], [13]. In this approach the time step size $\tau[n+1]$ is variable and at a $(n+1)$-th temporal level it is defined so that the ice layer thickness increases (or decreases) on a constant value h during this time step $\tau[n+1]$. Here h is a step of grid on spatial variable r. The value of the time step size $\tau\left[n+1\right]$ and the temperature distribution $T_i^{n+1}[j]$ in pipeline wall and in the ice layer are calculated by an iterative procedure at the every temporal level. The values $T_i^n[j]$ and $\tau\left[n\right]$ are assumed known. We use a uniform spatial grid $r_i\left[j\right] = R_{i-1} + jh, \ j = 0,1,...,N_i, \ i = 1,2,4$. We denote $T_i^{n+1}[j], \ i = 1,2,4$ the temperature at j-th grid node at $(n+1)$-th temporal level and $T_i^s[j], \ i = 1,2,4$ as temperature at j-th grid node at $(n+1)$-th temporal level on s-th iteration. Let τ^s is the time step size at the $(n+1)$-th temporal level on the s-th iteration. After the end of the iteration process the required value $\tau\left[n+1\right]$ is assigned the value τ^{s+1} of the last iteration. The overall process time is $t_{n+1} = t_0 + \tau[1] + \tau[2] + ... + \tau[n+1]$. The ice layer thickness at the moment $t = t_{n+1}$ is equal to $y = y_0 + (n+1)h.$

Let on the s-th iteration the time step size τ^s is set. Problem for the heat equations (13), (17), (20) with boundary conditions (15), (16), (19), (22) is solved by an implicit finite-difference scheme. Thus, the temperature in pipeline wall $T_i^s[j]$, $i = 1,2$ and in the ice layer $T_4^s[j]$ is calculated. The value of step size on next iteration τ^{s+1} is determined by following algorithm. Using the found sequence of the ice temperature $T_4^s[j], \ j = 1,...,N_4$ the heat fluxes are computed:

$$q_1 = \lambda_4\left(T_4^s[1] - T_4^s[0]\right)\Big/h, \tag{30}$$

$$q_2 = \lambda_4\left(T_4^s[N_4] - T_4^s[N_4-1]\right)\Big/h, \quad T_4^s[N_4] = T_*. \tag{31}$$

Then the thickness of the ice layer is obtained as:

$$h^s = \pm\tau^s\left(q_2 - q_3(T^*)\right)\Big/\tilde{Q}. \tag{32}$$

The value of the step size on next iteration is defined as:

$$\tau^{s+1} = \tau^s \pm \tilde{Q}\left(h - h^s\right)\Big/\left(q_1 - q_3(T^*)\right). \tag{33}$$

The iterative process is terminated if the inequality $\left|\tau^{s+1} - \tau^s\right| \le \varepsilon$ holds, where the value of ε is set in advance. In the case of the ice layer growing, if the glaciation conditions (24), (25) are fulfilled, in the right hand side of the equations (32), (33) we use the plus sign and the quantity $\tilde{Q} = \gamma\rho_4 + \alpha.$ In the case of the ice melting in the right hand side of the equations (32), (33) we use the negative sign and the quantity $\tilde{Q} = \gamma\rho_4.$

The first time step size in the zeroth approximation can be found, for example, using the known analytical Stefan problem solution [14]. For the following steps at the $(n+1)$-th temporal level as a zeroth approximation can choose the previous step size $\tau^0 = \tau[n]$.

3.2 The Admissibility Conditions of a Quasi-Stationary Heat Exchange Model

The admissibility conditions of a quasi-stationary heat exchange model for the considered regimes were specified. The characteristic rate W_x of growing ice of thickness y_x is equal to:

$$W_x = \frac{a_4}{y_x}, \quad a_4 = \frac{\lambda_4 \left(T_* - T\right)}{\gamma \rho_4}.$$

The characteristic rate u_4 of setting quasi-stationary temperature distribution in the ice of thickness y_x is equal to:

$$u_4 = \frac{k_4}{y_x}, \quad k_4 = \frac{\lambda_4}{\rho_4 c_4}.$$

The Stefan number $\mathrm{Ste} = c_4 \left(T_* - T\right)/\gamma$ can present as relation W_x to u_4. For the multilayer areas, the smallness condition of only the Stefan number is not sufficient. For considered regimes the following inequality holds

$$W_x \ll u_m, \quad u_m = \min\left(u_1, u_2, u_4\right), \ u_1 = k_1/\delta_1,$$
$$u_2 = k_2/\delta_2, \quad u_4 = k_4/y_x,$$

where $k_j = \lambda_j/\left(\rho_j c_j\right)$ is the temperature conductivity coefficient of j-th layer, δ_j is the thickness of j-th layer. In other words the smallness of all dimensionless groups $B_j = W_x/u_j$, $j = 1, 2, 4$ is necessary. From this inequality follows admissibility of a quasi-stationary approximation, this significantly simplifies computations of solution (1)-(23).

4 Boundary and Initial Conditions

We consider the unsteady problem of gas transportation, in which nonstationarity is due to the gas consumption variations and the ice formation processes.

4.1 Initial Conditions

For model (1)-(9) the initial conditions (8) are flow characteristics of steady regime:

$$t = 0: \quad \rho u = \mathrm{const} = \frac{W}{\pi R^2},$$
$$\rho(z) = \rho_0(z), \quad T(z) = T_0(z), \quad y(z) = y_0(z).$$

The functions $\rho_0(z)$, $T_0(z)$, $y_0(z)$ are calculated using the steady variant of this model presented in the book [8]. W is the mass flow rate, which is constant for the steady regime.

4.2 Boundary Conditions

The gas flow in the pipeline is subsonic. In the considered problem, unchanged over time, the inlet pressure and the inlet gas temperature are given. Using these values, the density and the internal energy are determined from the caloric equation and the equation of state. At outlet, the law of variation of the specific flow rate $w_*(t)$ is given.

Thus, the boundary conditions (9) are written this way:

$$z = 0: \ p(0,t) = p_0, \quad T(0,t) = T_0,$$
$$z = L: \ y(L,t) = w_*,$$

L is the length of gas pipeline.

Calculations were carried out for following parameters:

$W = 570$ kg s^{-1}, $T^* = 272.15$ K, $T_* = 271.24$ K,
$\alpha = 330048.81$ kJ m^{-3}, $L = 450$ km, $R = 0.5$ m,
$R_1 = 0.54$ m, $R_2 = 0.66$ m, $\gamma = 303000$ J kg^{-1},
$\tilde{c}_v = 1712.25$ J(kg K)$^{-1}$, $\lambda_1 = 24$ W(m K)$^{-1}$,
$\lambda_2 = 1.7$ W(m K)$^{-1}$, $\lambda_3 = 0.56$ W(m K)$^{-1}$,
$\lambda_4 = 2.15$ W(m K)$^{-1}$, $c_1 = 450$ J(kg K)$^{-1}$,
$c_2 = 924$ J(kg K)$^{-1}$, $c_3 = 4200$ J(kg K)$^{-1}$,
$c_4 = 2100$ J(kg K)$^{-1}$, $\rho_1 = 10000$ kg m^{-3},
$\rho_2 = 2300$ kg m^{-3}, $\rho_3 = 1005$ kg m^{-3},
$\rho_4 = 931$ kg m^{-3}, $\alpha_0 = 500$ W m^{-2}K^{-1},
$p(0,t) = 17.2$ MPa, $T(0,t) = 303.15$ K.

5 Numerical Simulations

Different approaches to a numerical solution of the general model equation system (1)-(23) were considered. An overview and comparison of different numerical techniques for the gas flow computation in pipelines can be found in number articles, for instance [15], [16]. Numerical solution to model (1)-(23) has been performed by using modified Lax-Wendroff scheme [17], which appeared to be preferable for the considered problems due to the count rate and the simplicity of implementation.

In conservative dimensionless form the model of gas transportation can be written as

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial z} = \Psi.$$

The vectors U, F, Ψ are given by

$$U = \begin{pmatrix} \rho \\ w \\ \rho\varepsilon + m_3 \dfrac{w^2}{\rho} \end{pmatrix}, \quad F = \begin{pmatrix} w \\ \dfrac{w^2}{\rho} + m_1 p \\ w\varepsilon + m_3 \dfrac{w^3}{\rho^2} + m_4 \dfrac{w}{\rho} p \end{pmatrix},$$

$$\Psi = \begin{pmatrix} 0 \\ -m_2 \dfrac{w^2}{\rho} \\ m_5 q \end{pmatrix}.$$

Where w is the flow rate $w = \rho u$; $m_1 - m_5$ are the dimensionless complexes, which expressed through the physical parameters and the characteristic values by the formulas

$$m_1 = \frac{p_x}{\rho_x u_x^2}, \quad m_2 = \frac{\lambda l_x}{4R}, \quad m_3 = \frac{u_x^2}{2\varepsilon_x},$$

$$m_4 = \frac{p_x}{\rho_x \varepsilon_x}, \quad m_5 = \frac{2\lambda t_x T_x}{\rho_x \varepsilon_x R r_x}.$$

Where p_x, ρ_x, u_x, T_x, ε_x are the characteristic pressure, density, velocity, temperature and internal energy of a real gas mix respectively; l_x, t_x are the characteristic length and time.

The algorithm consists of two steps. At every step the desired values of the density ρ, the flow rate w and the internal energy ε are determined explicitly:

stage 1

$$\frac{U_{k+1/2}^{n+1/2} - 0.5\left(U_k^n + U_{k+1}^n\right)}{0.5\tau} + \frac{F_{k+1}^n - F_k^n}{\Delta} = \Psi_{k+1/2}^n,$$

stage 2

$$\frac{U_k^{n+1} - U_k^n}{\tau} + \frac{F_{k+1/2}^{n+1/2} - F_{k-1/2}^{n+1/2}}{\Delta} = \Psi_k^{n+1/2}.$$

Where n, τ are a number and a time step size; Δ, k are a number and a space grid step size. In this scheme calculation of heat flux vector $q_k^{n+1/2}$ is performed by using the heat transfer model (13)-(23). As an illustration in Fig. 1 shows the ice growing dynamic on the outer surface of an offshore gas pipeline during five days, obtained from the numerical solution of model (1)-(23).

6 Results and Conclusions

Conducted research allowed the development of computational models of gas mix transportation through a long offshore gas pipeline in the northern seas.

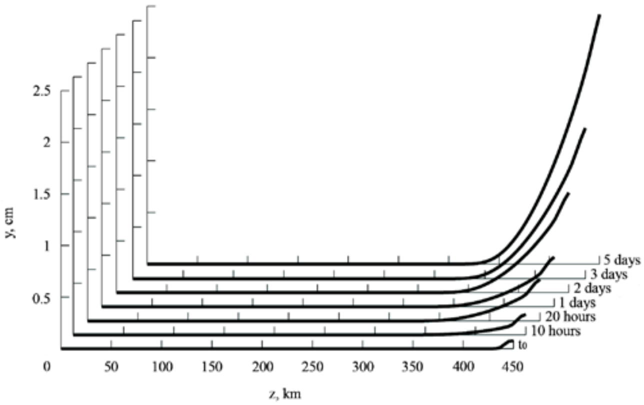

Figure 1. The ice growing dynamic on the outer surface of an offshore gas pipeline during five days.

This model spans a wide variety of the practical problems. Based on numerical modelling, this research led to numerous practical suggestions for selecting the transportation regime, together with the gas pipeline construction and selection of the gas pipeline route relief. These include:

1. A pipeline route with a sharp rise and a gentle slope conforming to the requirements of the technological constraints) is hydraulically preferable to a route of the same length, but with a long sloping rise and an abrupt slope.
2. The computational model provides a way to compute an admitted gas pressure and temperature region at inlet, at which the flow velocity will not achieve a specified rate limit. As known, beginning from this critical velocity the pipeline vibration occurs.
3. The computational model includes accounting for influence of the ambient temperature, the seawater salinity, and the transition temperature on the dynamic of the ice grow. It is adequate to calculate a thickness of the ice layer upon the outer surface of the gas pipeline, and important to assess the pipeline buoyancy.

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