Ferrofluid flow along stretched surface under the action of magnetic dipole

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Abstract: - The aim of this paper is to present numerical investigation on the magneto-thermomechanical interaction between heated viscous incompressible ferrofluid and a cold wall in the presence of a spatially varying magnetic field. The two-dimensional parallel flow of a heated saturated ferrofluid along a surface, whose temperature increases linearly with distance from the leading edge, under the influence of the magnetic field due to two equally directed line currents which are perpendicular to the flow plane and equidistant from the wall is investigated. The influence of governing parameters corresponding to various physical conditions are analyzed. Numerical results are exhibited for the distributions of velocity and temperature, and the effect of the Prandtl number, the power law exponent and the ferrohydrodynamic interaction parameter is presented.

Key-Words: ferrofluid, magnetic field, boundary layer, similarity transformation, magnetic dipole

1 Introduction

The nanofluid is a homogenous combination of base fluid and nanoparticles. These suspensions are made of various metals or non-metals e.g., aluminum (Al), copper (Cu), Silver (Ag), and graphite or carbon nanotubes respectively, and the base fluid, which includes water, oil or ethylene glycol. Ferrofluid is a special type of nanofluids, liquids in which magnetic nanoparticles are suspended in single domain and carrier liquid. The base fluids for the ferrofluids are usually taken to be an oil or water. These fluids are strongly magnetized by an external magnetic field. Ferromagnetic fluid and the flow of energy transport can be control owing to external magnetic field. This fluid has attracted many researchers and scientists because it has uncountable applications in our daily lives and technological processes. Such type of applications includes heat exchanger, vehicle cooling, nuclear reactor, cooling of electronic devices.

In the recent years, boundary layer flow and energy transport phenomena due to stretched surface have conquered significant importance because of wider range of applications in engineering and modern industrial processes. These are continuous stretching of plastic films, polymer extrusion, glass blowing, tinning and annealing of copper wires, chemical industries such as metallurgy process like metal extrusion and metal spinning, heat removal from nuclear fuel debris, artificial fibres, hot rolling and so on.

Ferrofluids are applicable in enhancing the heat transfer rate in several materials and liquids used in advanced technology and industry. It plays an important role in the field of chemical and electromechanical devices. Stephen [1] originated ferromagnetic liquids. Andersson and Valnes [2] investigated heat transfer rate in ferromagnetic fluids. The effects of magnetic fields and thermal gradients are explored by Neuringer [3]. Albrecht et al. [4] exposed domains for the ferromagnetism and ferromagnetic effects in liquids. The forced and free convective boundary layer flow of a magnetic fluid over a semi-infinite vertical plate, under the action of a localized magnetic field, was numerically studied by Tzirtzilakis et al. [5].

When magnetizable materials are subjected to an external magnetizing field **H**, the magnetic dipoles or line currents in the material will align and create a magnetization **M**.

Problem of magnetohydrodynamic (MHD) flow near infinite plate has been studied intensively by a number of investigators. The influence of magnetic dipole in a non-Newtonian ferrofluid was characterized for an incompressible stretchable cylinder by Awais et al. [6]. The boundary layer heat transport flow of multiphase magnetic fluid past a stretching sheet under the impact of circular magnetic field was described in [7]. Numerical investigation of magnetohydrodynamic Sisko fluid flow over linearly stretching cylinder along with combined effects of temperature depending thermal conductivity and viscous dissipation was presented by Hussain et al. [8].

Neuringer [3] has investigated numerically the dynamic response of ferrofluids to the application of non-uniform magnetic fields with studying the effect of magnetic field on two cases, the two-dimensional stagnation point flow of a heated ferrofluid against a cold wall and the two-dimensional parallel flow of a heated ferrofluid along a wall with linearly decreasing surface temperature.

The aim of this paper is investigating the static behaviour of ferrofluids along a wall with linearly increasing surface temperature in magnetic fields applying similarity method. Numerical computations are accomplished, and interesting aspects of flow velocity and temperature are exhibited for different parametric conditions. Wall shear stress, heat transfer, velocity and temperature boundary layer profiles are obtained and compared with the results obtained in [3].

2 Problem Formulation

Consider a steady two-dimensional flow of an incompressible, viscous and electrically nonconducting ferromagnetic fluid over a flat surface in the horizontal direction seen in Fig. 1.
The two magnetic dipoles are equidistant a from the leading edge. The filed is due to two-line currents perpendicular to and directed out of the flow plane.

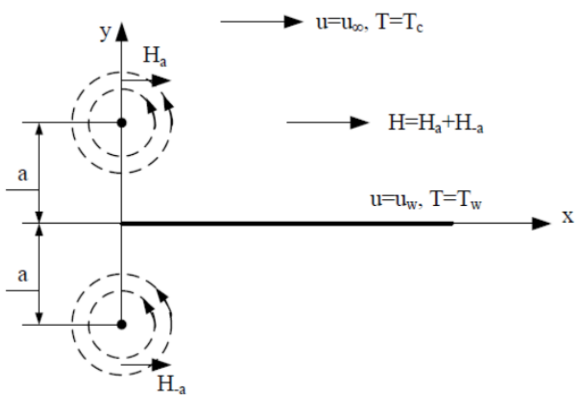

Fig. 1. Parallel flow along a flat surface in magnetic field

The existence of spatially varying fields is required in ferrohydrodynamic interactions [9].
The following assumptions are needed:
(i) the direction of magnetization of a fluid element is always in the direction of the local magnetic field,
(ii) the fluid is electrically non-conducting and
(iii) the displacement current is negligible.

Introducing the magnetic scalar potential ϕ whose negative gradient equals the applied magnetic field, i.e. $\mathbf{H} = -\nabla\phi$, the scalar potential can be given by

$$\phi(x,y) = -\frac{I_0}{2\pi}\left(tan^{-1}\frac{y+a}{x} + tan^{-1}\frac{y-a}{x}\right),$$

where I_0 denotes the dipole moment per unit length and a is the distance of the line current from the leading edge.

In the boundary layer for regions close to the wall when distances from the leading edge large compared to the distances of the line sources from the plate, i.e. $x \gg a$, then one gets

$$[\nabla H]_x = -\frac{I_0}{\pi}\frac{1}{x^2}, \quad (1)$$

where H is the magnetic field.

The boundary layer equations for a two-dimensional and incompressible flow are based on expressing the conservation of mass, continuity, momentum and energy.

The analysis is based on the following four assumptions [3]:

(i) the applied field is of sufficient strength to saturate the ferrofluid everywhere inside the boundary layer,

(ii) within the temperature extremes experienced by the fluid, the variation of magnetization with temperature can be approximated by a linear equation of state, the dependence of M on the temperature T is described by $M = K(T_C\text{-}T)$, where K is the pyromagnetic coefficient and T_c denotes the Curie temperature as proposed in [3, 10],

(iii) the induced field resulting from the induced magnetization compared to the applied field is neglected; hence, the uncoupling of the ferrohydrodynamic equations from the electromagnetic equations and

(iv) in the temperature range to be considered, the thermal heat capacity c, the thermal conductivity k, and the coefficient of viscosity ν are independent of temperature.

The governing equations are described as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\ , \qquad (2)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{I_0\mu_0 k}{\pi\rho}(T_C-T)\frac{1}{x^2}+\nu\frac{\partial^2 u}{\partial y^2}\ , \qquad (3)$$

$$c\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=k\frac{\partial^2 T}{\partial y^2}, \qquad (4)$$

where u and v are the parallel and normal velocity components to the plate, the x and y axes are taken parallel and perpendicular to the plate, respectively, ν is the kinematic viscosity and ρ denotes the density of the ambient fluid, which will be assumed constant. The system (2)-(4) of nonlinear partial differential equations is considered under the boundary conditions at the surface $(y=0)$

$$u(x,0)=0,\ \ v(x,0)=0,\ T(x,0)=T_w \qquad (5)$$

with increasing temperature at the wall $T_w=T_C+Ax^{n+1}$ and

$$u(x,y)\to u_\infty,\ T(x,y)\to T_\infty \qquad (6)$$

as y leaves the boundary layer $(y\to\infty)$ with $T_\infty=T_c$, and u_∞ is the exterior streaming speed which is assumed throughout the paper to be $u_\infty=U_\infty x^n$ (with constant U_∞). Parameter m is relating to the power law exponent. The parameter $n=0$ refers to a linear temperature profile and constant exterior streaming speed. In case of $n=1$, the temperature profile is quadratic, and the streaming speed is linear. The value of $n=-1$ corresponds to no temperature variation on the surface.

Introducing the stream function ψ, defined by $u=\partial\psi\,/\,\partial y$ and $v=-\partial\psi\,/\,\partial y$, so equation (2) is automatically satisfied, and equations (3) – (4) can be formulated as

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\Psi}{\partial yx}-\frac{\partial\Psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\nu\frac{\partial^3\Psi}{\partial y^3}-\frac{I_0\mu_0 K}{\pi\rho}(T_c-T), \qquad (7)$$

$$c\left[\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}\right]=k\frac{\partial^2 T}{\partial y^2}\,. \qquad (8)$$

Boundary conditions (5) and (6) are transformed to

$$\psi_y'(x,0)=0,\ \ \psi_x'(x,0)=0,\ \ \ T(x,0)=T_C+Ax^{n+1}, \qquad (9)$$

$$\psi_y'(x,y)\to U_\infty x^n,\ \ T(x,y)=T_c \text{ as } y\to\infty. \qquad (10)$$

Now, we have two single unknown functions and two partial differential equations. The system of (8)–(10) allows us to look for similarity solutions of a class of solutions ψ and T in the form (see [11, 12])

$$\left.\begin{array}{c}\psi(x,y)=Bx^b f(\eta)\\ T=T_C+Ax^{n+1}\Theta(\eta)\\ \eta=Cx^d y\end{array}\right\} \qquad (11)$$

where b and d satisfy the scaling relation $b+d=n$ and for coefficients B and C the relation $B\,/\,C=\nu$ must be fulfilled. The real numbers b, d are such that $b-d=1$ and $BC=U_\infty$, i.e.

$$b=\frac{n+1}{2},\ \ d=\frac{n-1}{2}$$

$$B=\sqrt{\nu U_\infty},\ \ C=\sqrt{\frac{U_\infty}{\nu}}\,.$$

By considering (11), equations (7) and (8) and conditions (9) and (10) lead to the following system of coupled ordinary differential equations

$$f'''-nf'^2+\frac{n+1}{2}ff'-\beta\Theta=0, \qquad (12)$$

$$\Theta''+(n+1)Pr\left(\frac{1}{2}f\Theta'-\Theta f'\right)=0 \qquad (13)$$

subjected to the boundary conditions

$$f(0)=0,\ \ f'(0)=0,\ \ \Theta(0)=-1 \qquad (14)$$

$$f'(\eta)=1,\ \ \Theta(\eta)=0\ \ as\ \ \eta\to\infty \qquad (15)$$

where $Pr=c\nu\,/\,k$ is the Prandtl number and $\beta=I_0\mu_0 KA/\pi\rho U_\infty^2$ is the ferrohydrodynamic interaction parameter.

The components of the non-dimensional velocity $\vec{v}=(u,v,0)$ can be expressed by

$$u=U_\infty x^n f'(\eta),$$

$$v=-\sqrt{\nu U_\infty}\,x^{\frac{n-1}{2}}\left(\frac{n+1}{2}f(\eta)+\frac{n-1}{2}f'(\eta)\eta\right).$$

The shear stress and the heat transfer at the wall are derived by the drag coefficient $f''(0)$ and the $\Theta'(0)$.

If $n=0$ and $\beta=0$, equation (12) is equivalent to the well-known Blasius equation

$$f'''+\frac{1}{2}ff'=0 \qquad (16)$$

which appears when analysing a laminar boundary-layer problem for Newtonian fluids [12, 14].

During our investigation we suppose that the distances x are greater than a.
Moreover, the fluid is nonelectrically conducting. The model describes the dynamics of heat transfer in an incompressible magnetic fluid under the action of an applied magnetic field.

3 Numerical Solution

There are several proceedings for the numerical solution of boundary value problems of coupled strongly nonlinear differential equations as (12)-(13). We use the higher derivative method (HDM) for the boundary value problem (12)–(15), which is implemented in Maple. This method is applicable in the numerical analysis of some boundary value problems and ensure the stability by using higher derivatives [15].

The setting of digits in our case is digits:=15. The boundary value problem is considered as a first order system, where

$y1(x) = f(\eta)$,
$y2(x) = f'(\eta)$,
$y3(x) = f''(\eta)$,
$y4(x) = \Theta(\eta)$,
$y5(x) = \Theta'(\eta)$.

The left and right boundary conditions are defined by bc1 and bc2. It is necessary to give the range (bc1 to bc2) of the boundary value problem (Range:= $[0.0, \eta_{max}]$). We have three parameters, n, β and Pr (e.g., pars:= n=0.0, β =0.0, Pr=10.0).

The next step is to define the initial derivative in nder and the number of the nodes in nele (nder:= 3; nele := 5;). Next settings of the absolute and relative tolerance for the local error are (atol:= 1e-6; rtol:= atol /100;). The HDMadapt procedure is applied to determine the approximate numeric solution. The simulation gives the figure of all solution functions (from y1 to y5).

4 Results and Conclusion

The numerical examination of a heated ferrofluid flow in magnetic field over a flat surface with boundary conditions is studied.

The arising set of flow govern equations are simplified under usual boundary layer assumptions. A set of variable similarity transforms are employed to shift the governing partial differential equations into ordinary differential equations. e solution of attained highly nonlinear simultaneous equations is computed by an efficient technique with HDM method. Numerical computations are executed, and different aspects of flow velocity and temperature are exhibited on Figs. 2-6 for different parametric conditions.

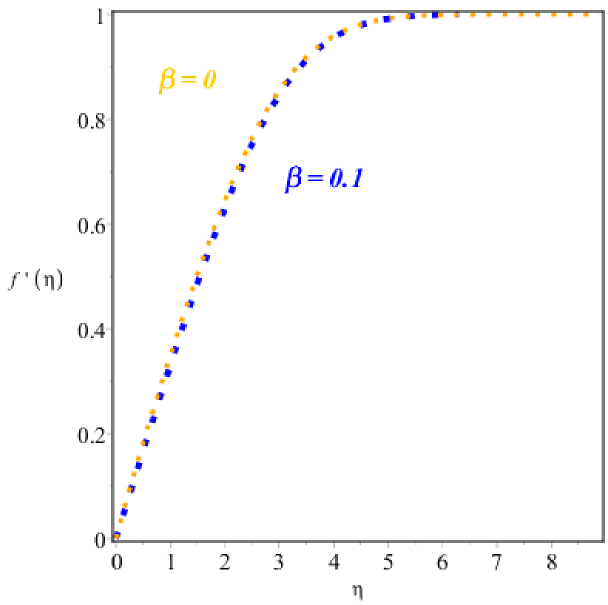

Figure 2. The velocity distribution (Pr=10, n=0, β=0 and β=0.1)

Figures 2-6. show the effect of parameter β for the velocity and thermal distributions. If the parameter value β increases, then the boundary layer thickness increases for the thermal distribution solutions. Significant impact on the velocity distribution of β cannot be noticed for n=0. An opposite effect can be seen in case of increasing Prandtl number for the temperature distribution.

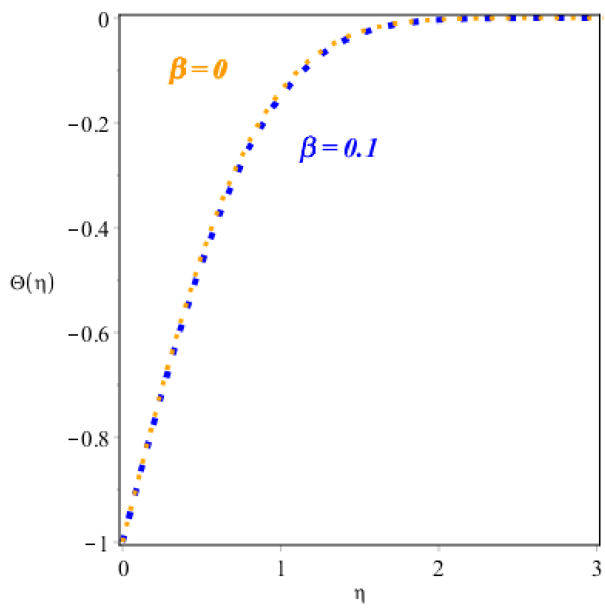

Figure 3. The thermal distribution (Pr=10, n=0, β=0 and β=0.1)

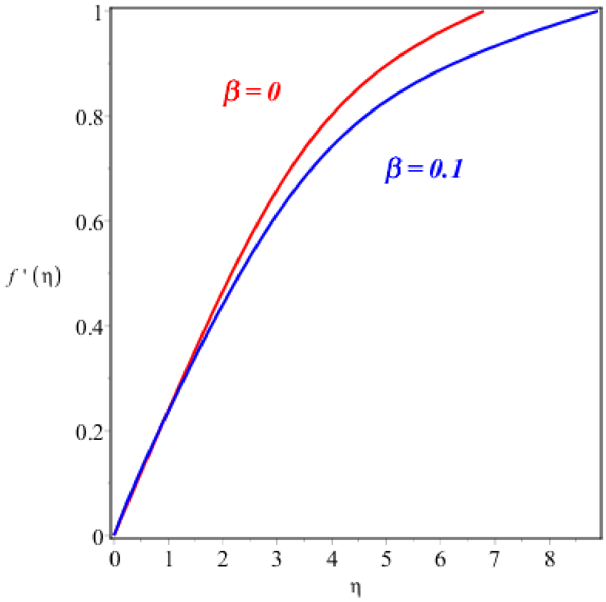

Figure 4. The velocity distribution for different values of β (Pr=10, n=0.1) for $\beta = 0$, and $\beta = 0.1$

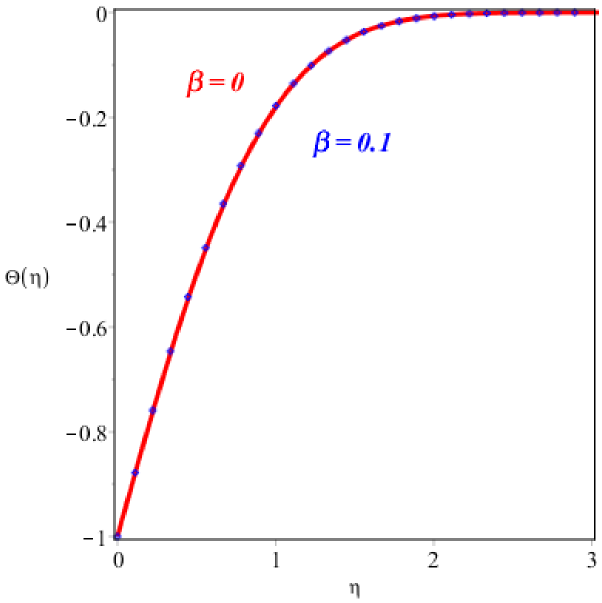

Figure 5. The temperature distribution for different values of β (Pr=10, n=0.1) for $\beta = 0$, and $\beta = 0.1$

For n=0.1 one can observe, that there is no effect of β on the temperature distribution, while for increasing β the boundary layer thickness is also increasing. Additionally, the parameters involved in the boundary value problem influence the coefficient of skin friction and influence the flow parameters on wall shear stress.

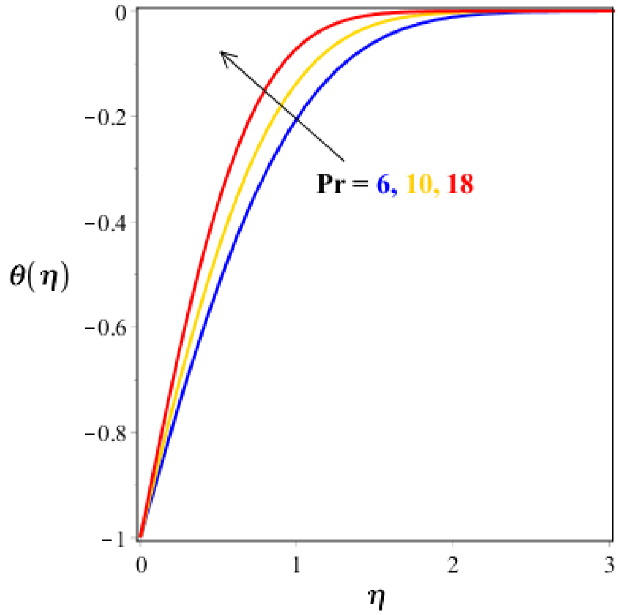

Figure 6. The temperature distribution for different values of Pr ($\beta = 0, n$=0)

Acknowledgements:

Project no. 129257 has been implemented with the support provided from the National Research, Development and Innovation Fund of Hungary, financed under the K_18 funding scheme.
The described article was carried out as part of the EFOP-3.6.1-16-2016-00011 “Younger and Renewing University – Innovative Knowledge City – institutional development of the University of Miskolc aiming at intelligent specialisation” project implemented in the framework of the Szechenyi 2020 program. The realization of this project is supported by the European Union, co-financed by the European Social Fund.”

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