Distribution of the Maximum Costs of Products in Direct Selling

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Abstract: Direct selling companies do not reveal the cost of their products and typically design complex rules for their employees regarding commission. However, knowing the product costs may assist us in selecting an appropriate choice from among the numerous direct selling companies. In this study, statistical knowledge is used to develop a binomial-geometric model for the complicated commission rules. Using this model, the probability distribution is proved and the mean and standard deviation percentages of cost are found for all employees according to the employee proportions in all stages of the two examples. Employees in lower stages must pay more commission to multilevel above employees, particularly in companies with few top employees. Obtaining sufficient employees to earn commission from the low-cost direct selling product is challenging. Therefore, employees should individually judge whether the direct selling products are worth purchasing and selling.

Key–Words: Applied probability, Binomial-geometric distribution, Direct selling, Upgrading and commission rules, Proportion of commission, Distribution of maximum product cost

1 Literature Review

1.1 Research Scope

Vander Nat and Keep (2002) mentioned that a specific form of direct selling, multilevel-marketing (MLM), experienced significant international growth during the 1990s. A corresponding increase in the investigation and prosecution of illegal pyramid schemes occurred during the same period.

Stanger (2008) reported that the direct selling industry has a long history. However, the industrys reputation has suffered from the prevalence of misleading and illegal schemes, such as pyramid schemes, which generate income by paying money or other compensation for the sole act of recruiting, charge high entrance fees, and sell products of questionable value.

Kron (2009) stated that the advantages of direct setting for consumers include good information provision, quality of service, relationship building, and convenience. However, the potential for harm also exists due to the limited knowledge of direct setters and excessive selling hype.

However, numerous people still have many doubts regarding direct selling despite its long history. Therefore, this study will investigate a legal direct selling company, and establish a standard for detecting potential risks.

Table 1: The commission of recruiting routes for form C1

TOP +T0.5%	TOP +T0.5%	TOP +T0.5%	TOP +T0.5%	TOP +T0.5%
C_{11} -0.5%	M +$G$10%	M +$N$5%	M +$N$2%	M +$N$1%
	C_{12} -10.5%	M +$G$10%	M +$N$5%	M +$N$2%
		C_{13} -15.5%	M +$G$10%	M +$N$5%
			C_{14} -17.5%	M +$G$10%
				C_{15} -18.5%

1.2 Research Aim

Homburg, et al. (2009) followed research of social cognition and introduced the concept of customer need knowledge (CNK), which describes the extent to which a frontline employee can accurately identify a customers hierarchy of needs.

The characteristic that every customer of the direct selling company is also an employee must also be investigated. Additionally, the customers or employees should at least attempt to understand the costs of the products they are selling.

1.3 Research Method

Iyer (2002) reported that an order production lead time affects the inventory costs at retail locations. He

also examined the impact of changing from a first-come first-served (FCFS) production rule for orders arriving at the production facility to a rule providing non-preemptive priority (PR) to orders from retail locations with higher demand uncertainty. Three approximations for the ratio of inventory costs under PR and FCFS were provided and then used to identify conditions where PR offers a greater decrease to retail inventory costs compared to FCFS.

Jayasree and Swamy (2006) observed that numerous real-life phenomena are represented by power series distributions (PSD), such as Poisson, negative binomial, and geometric. Based on these models, a number of new probability distributions were obtained.

Legara, et al. (2008) analyzed two multi-level marketing enterprise (MLM) network architecture types, unilevel and binary, regarding growth behavior and earning potential among members. They found that unilevel MLM's do not exhibit the Pareto earning distribution, and earning potential is independent of member position in the network.

The offering distribution, which is the opposite of the earning distribution, is described in this paper. This study derives a new statistical model, which combines binomial and geometric distributions for the offering distribution, using the known proportions or ratios of employees in all stages.

2 Introduction

The upgrading and commission rules of a number of direct selling companies are simplified to develop an easier model, but they can be extended to a complete model in the future. A company possesses four employee stages: the consultant (C), the manager (M), the area-manager (AM), and the top (TOP). Any person who has paid the start-up of TWD$1,300 (approximately USD$45) becomes a C, the lowest stage of employees. Under certain conditions a C will be upgraded to M, and then M will be upgraded to AM. Finally, AM will be upgraded to TOP, the highest stage of employees.

To understand the upgrading and commission rules, the following terminologies require explanation.

1. *Level*:Any employee is 1 level below the employee who recruited them to the company. For example, in Table 1, C_{11} is 1 level below TOP and TOP is 1 level above C_{11}. C_{12} is 2 levels below TOP and TOP is 2 levels above C_{12}. Generally, employees who joined the company earlier are in the higher levels. However, comparing the levels

Table 2: The commission of recruiting routes for form C2

TOP +T0.5%	TOP +T0.5%	TOP +T0.5%	TOP +T0.5%
M (∞)	AM +$N$5%	AM +$N$2%	AM +$N$1%
AM +$G$15% -$N$8%	AM +$G$15% -$N$8%	AM +$N$5%	AM +$N$2%
M +$N$1%	M +$N$1%	AM +$G$15% -$N$8%	AM +$N$5%
M +$N$2%	M +$N$2%	M +$N$1%	AM +$G$15% -$N$8%
M +$N$5%	M +$N$5%	M +$N$2%	M +$N$1%
M +$G$10%	M +$G$10%	M +$N$5%	M +$N$2%
C_{21} -25.5%	C_{22} -30.5%	M +$G$10%	M +$N$5%
		C_{23} -32.5%	M +$G$10%
			C_{24} -33.5%

Table 3: The commission of recruiting routes for form C3

TOP +T0.5%	TOP +T0.5%	TOP +T0.5%	TOP +T0.5%
M (∞)	AM +$N$5%	AM +$N$2%	AM +$N$1%
AM +$G$15% -$N$7%	M (∞)	AM +$N$5%	AM +$N$2%
M +$N$2%	AM +$G$15% -$N$7%	M (∞)	AM +$N$5%
M +$N$5%	M +$N$2%	AM +$G$15% -$N$7%	M (∞)
M +$G$10%	M +$N$5%	M +$N$2%	AM +$G$15% -$N$7%
C_{31} -25.5%	M +$G$10%	M +$N$5%	M +$N$2%
	C_{32} -30.5%	M +$G$10%	M +$N$5%
		C_{33} -32.5%	M +$G$10%
			C_{34} -33.5%

of employees under different recruiting routes is meaningless because they cannot earn commission from each other. For example, in Table 1, the levels of C_{11} and C_{12} should not be compared because they joined via different recruiting routes.

2. *Group*: The leader and all employees with stages below the leader comprise a group. In Table 1 for example, the $TOP - C_{11}$ recruiting route is part of a TOP -group. The $TOP - M - C_{12}$ recruiting route is also part of a TOP -group. However, $M - C_{12}$ is typically considered part of an M -subgroup. A group does not include subgroups led by employees who are in a higher stage than the original group leader is. For example, in Table 6, the $TOP - M(\infty) - AM - C_{61}$ recruiting route is part of a TOP -group and the $AM - C_{61}$ recruiting route is part of an AM -subgroup, but none of the infinite M -subgroups should include the AM -subgroup.

3. *Direct employee*: An employee who is 1 level below the group leader

4. *Generation*: One group or subgroup is counted as one generation. The first-generation subgroup is a subgroup of the original group, the second-generation subgroup is a subgroup of the first-generation subgroup, and the third-generation subgroup is a subgroup of the second-generation subgroup. In Table 1 for example, a recruiting route can be found from the TOP to the first-generation M -subgroup, to the second-generation M -subgroup, to the third-generation M -subgroup, and to C_{14}.

5. TWD$: New Taiwan dollar

6. USD$: United States dollar

The upgrading rule of a company is as follows:

1. If C and all consultants in 1 to 3 levels below C together purchase products worth at least TWD$150,000 (approximately USD$5172) in three months, then C will be upgraded to M.

2. If M recruits at least two direct managers, and the M -group, including the leading M and three generations of M -subgroups, together purchase products worth at least TWD$600,000 (approximately USD$20,690) in three months, then the leading M will be upgraded to AM.

Table 4: The commission of recruiting routes for form C4

TOP +T0.5%	TOP +T0.5%	TOP +T0.5%	TOP +T0.5%
M (∞)	M (∞)	M (∞)	M (∞)
AM +$G$15% -$N$5%	AM +$N$5%	AM +$N$2%	AM +$N$1%
M +$N$5%	M (∞)	AM +$N$5%	AM +$N$2%
M +$G$10%	AM +$G$15% -$N$5%	M (∞)	AM +$N$5%
C_{41} -25.5%	M +$N$5%	AM +$G$15% -$N$5%	M (∞)
	M +$G$10%	M +$N$5%	AM +$G$15% -$N$5%
	C_{42} -30.5%	M +$G$10%	M +$N$5%
		C_{43} -32.5%	M +$G$10%
			C_{44} -33.5%

3. If AM recruits at least five direct area managers, and the AM -group including the leading AM and three generations of AM -subgroups, together purchase products worth at least TWD$6,000,000 (approximately USD$206,897) in one month, then the leading AM will be upgraded to TOP.

According to these upgrading rules, employees in higher levels are not necessarily also in higher stages. The stage of an employee depends on the amount of products purchased, not when they joined the company.

The company directly deposits the earned commission into the employee's bank account. The commission rule for each stage of employees is as follows:

1. For C: 16% rebate of their purchases

2. For M:

   (a) 16% rebate of their purchases

   (b) Group commission (+$G$10%): 10% commission from purchases by the leading M and consultants in the M -group

   (c) Nurture commission:

i. 5% commission from purchases by employees of first-generation M -subgroup(s) (+$N5\%$)

ii. 2% commission from purchases by employees of second-generation M -subgroup(s) (+$N2\%$)

iii. 1% commission from purchases by employees of third-generation M -subgroup(s) (+$N1\%$)

3. For AM:

(a) 16% rebate of their purchases

(b) Group commission:

i. 25% commission from purchases by the leading AM and consultants in the AM -group (+$G25\%$)

ii. 15% commission from purchases by employees of the direct M -subgroup(s) (+$G15\%$)

iii. Deduct the nurture commission for an M -subgroup to another M -subgroup(s)

(c) Nurture commission:

i. 5% commission from purchases by employees of first-generation AM -subgroup(s) (+$N5\%$)

ii. 2% commission from purchases by employees of second-generation AM -subgroup(s) (+$N2\%$)

iii. 1% commission from purchases by employees of third-generation AM -subgroup(s) (+$N1\%$)

4. For TOP:

(a) 16% rebate of their purchases

(b) Top commission (+$T0.5\%$): 0.5% commission from purchases by all employees in the TOP -group

3 The Model

Assume that the employee proportions of C, M, AM, and TOP in the company are R_C, R_M, R_{AM} and R_{TOP}, respectively. $R_C + R_M + R_{AM} + R_{TOP} = 1$. Because a consultant cannot obtain commission from other employees, the following probabilities are the only concerns: $P_M = R_M/(1 - R_C)$, $P_{AM} = R_{AM}/(1 - R_C)$, $P_{TOP} = R_{TOP}/(1 - R_C)$, with $P_M + P_{AM} + P_{TOP} = 1$.

For any of the recruiting routes C_{ji}, M_{ji}, or AM_i in Tables 1 to 12, all the leaders of the above group generations are traceable to TOP. Assuming that the above X_{th} generation of group leader is TOP, then in the preceding $X-1$ group leaders, AM appears N times, where $N$is an integer and $0 \leq N \leq X-1$. The joint distribution of X and N should be called a binomial-geometric distribution because the probability mass function is

Table 5: The commission of recruiting routes for form C5

TOP +$T0.5\%$	TOP +$T0.5\%$	TOP +$T0.5\%$	TOP +$T0.5\%$
M (∞)	M (∞)	M (∞)	M (∞)
AM +$G15\%$	AM +$N5\%$	AM +$N2\%$	AM +$N1\%$
M +$G10\%$	AM +$G15\%$	AM +$N5\%$	AM +$N2\%$
C_{51} -25.5%	M +$G10\%$	AM +$G15\%$	AM +$N5\%$
	C_{52} -30.5%	M +$G10\%$	AM +$G15\%$
		C_{53} -32.5%	M +$G10\%$
			C_{54} -33.5%

$$P(x,n) = \frac{(x-1)!}{n!(x-1-n)!} P_M^{x-1-n} P_{AM}^n P_{TOP} \quad (1)$$

The marginal probability mass function for X is a geometric probability mass function:

$$P_X(x) = \sum_{n=0}^{x-1} \frac{(x-1)!}{n!(x-1-n)!} P_M^{x-1-n} P_{AM}^n P_{TOP} = (P_M + P_{AM})^{x-1} P_{TOP} = P_{TOP}(1-P_{TOP})^{x-1} \quad (2)$$

with $E(X) = \frac{1}{P_{TOP}}$, $Var(X) = \frac{1-P_{TOP}}{P_{TOP}^2}$, and $M_X(t) = \frac{P_{TOP} \cdot e^t}{1-(1-P_{TOP})e^t}$.

Similar recruiting routes should be combined to develop a form. According to the AM form in Table 12, the marginal probability mass function for N is also a geometric probability mass function:

$$P_N(n) = \sum_{x=1}^{\infty} \frac{(x-1)!}{n!(x-1-n)!} P_M^{x-1-n} P_{AM}^n P_{TOP} = P(AM_n) = \frac{1}{1-P_M} \left(\frac{P_{AM}}{1-P_M}\right)^{n-1} P_{TOP} \quad (3)$$

Table 6: The commission of recruiting routes for form C6

TOP +T0.5%	TOP +T0.5%	TOP +T0.5%	TOP +T0.5%
M (∞)	M (∞)	M (∞)	M (∞)
AM +G25%	AM +N5%	AM +N2%	AM +N1%
C_{61} -25.5%	M (∞)	M (∞)	M (∞)
	AM +G25%	AM +N5%	AM +N2%
	C_{62} -30.5%	M (∞)	M (∞)
		AM +G25%	AM +N5%
		C_{63} -32.5%	M (∞)
			AM +G25%
			C_{64} -33.5%

with $E(N) = \frac{1-P_M}{P_{TOP}}$, $Var(N) = \frac{P_{AM}P_{TOP}}{(1-P_M)^3}$, and $M_N(t) = \frac{P_{TOP}}{1-P_M}e^t/[1 - \frac{P_{AM}}{1-P_M}e^t]$.

If the company has additional stages, the joint distribution becomes a multinomial-geometric distribution.

Commission is a function of X and N. For example, if N_j is the nurture commission and T is the top commission, then the commission for an AM to offer is,

$$B_{AM} = \begin{cases} \sum_{j=1}^{n} N_j + T & \text{if } n \leq 3 \\ \sum_{j=1}^{3} N_j + T & \text{if } n > 3 \end{cases}$$

Evidently, no close form exists for $E(B_{AM})$. The commission is computed following the same steps used to derive the marginal probability mass function for N. That is, similar experimental results are combined to form a recruiting route, and then similar recruiting routes are combined to establish a form.

4 Case Study

The recruiting routes C_{ji}, M_{ji} and AM_i are displayed to show how much commission an employee provides to other multilevel above employees. The T, G, and N percentages shown in the second or the third row of each cell in Tables 1 to 12 represent the top commission, group commission, and nurture commission, respectively.

An employee in a lower stage with a higher level cannot obtain commission from employee in a higher stage with a lower level.

The different forms are explained as follows:

C1 : No AM exists between C_{1i} and TOP.

C2 : Four adjacent Ms are directly above C_{2i}.

C3 : Three adjacent Ms are directly above C_{3i}.

C4 : Two adjacent Ms are directly above C_{4i}.

C5 : One M is directly above C_{5i}.

C6 : One AM is directly above C_{6i}.

M1 : No AM exists between M_{1i} and TOP.

M2 : One M is directly above M_{2i}.

M3 : Two adjacent Ms are directly above M_{3i}.

M4 : Three adjacent Ms are directly above M_{4i}.

M5 : One AM is directly above M_{5i}.

AM : Includes all employees between AM_i and TOP.

For the recruiting route C_{21} in Table 2, the leading M of the M -group containing the consultant C_{21} obtains a group commission of 10%; The leading manager of the above first-generation M -subgroup obtains a nurture commission of 5%; the leading manager of the above second-generation M -subgroup obtains a nurture commission of 2%; and the leading manager of the above third-generation M -subgroup obtains a nurture commission of 1%. Then, the leading AM of the AM -group containing C_{21} obtains a group commission of 15%, minus a total nurture commission of 8%.

5 Probability Distribution

Between the AM group leader and TOP group leader of the recruiting route C_{21} in Table 2, infinite group leaders could exist in stage M, and they would not obtain any commission from C_{21} based on the commission rules. Between the above fourth-generation M -group and the AM group leader of C_{21}, infinite group leaders, who do not obtain any commission from C_{21}, could also exist in stage M. That is the reason the term $1/(1 - P_M)$ appears.

The probabilities of recruiting routes for form C1 are as follows:

- $P(C_{11}) = P_{TOP}$
- $P(C_{12}) = P_M P_{TOP}$
- $P(C_{13}) = P_M^2 P_{TOP}$
- $P(C_{14}) = P_M^3 P_{TOP}$
- $P(C_{15}) = P_M^4(\sum_{h=0}^{\infty} P_M^h)P_{TOP} = P_M^4 P_{TOP}/(1 - P_M)$

The probabilities of recruiting routes for form C2 are as follows:

Table 7: The commission of recruiting routes for form M1

TOP +T0.5%	TOP +T0.5%	TOP +T0.5%	TOP +T0.5%
M_{11} -0.5%	M +$N$5%	M +$N$2%	M (∞)
	M_{12} -5.5%	M +$N$5%	M +$N$1%
		M_{13} -7.5%	M +$N$2%
			M +$N$5%
			M_{14} -8.5%

Table 8: The commission of recruiting routes for form M2

TOP +T0.5%	TOP +T0.5%	TOP +T0.5%	TOP +T0.5%
M (∞)	M (∞)	M (∞)	M (∞)
AM +$G$15% -$N$5%	AM +$N$5%	AM +$N$2%	AM +$N$1%
M +$N$5%	M (∞)	AM +$N$5%	AM +$N$2%
M_{21} -15.5%	AM +$G$15% -$N$5%	M (∞)	AM +$N$5%
	M +$N$5%	AM +$G$15% -$N$5%	M (∞)
	M_{22} -20.5%	M +$N$5%	AM +$G$15% -$N$5%
		M_{23} -22.5%	M +$N$5%
			M_{24} -23.5%

Table 9: The commission of recruiting routes for form M3

TOP +T0.5%	TOP +T0.5%	TOP +T0.5%	TOP +T0.5%
M (∞)	M (∞)	M (∞)	M (∞)
AM +$G$15% -$N$7%	AM +$N$5%	AM +$N$2%	AM +$N$1%
M +$N$2%	M (∞)	AM +$N$5%	AM +$N$2%
M +$N$5%	AM +$G$15% -$N$7%	M (∞)	AM +$N$5%
M_{31} -15.5%	M +$N$2%	AM +$G$15% -$N$7%	M (∞)
	M +$N$5%	M +$N$2%	AM +$G$15% -$N$7%
	M_{32} -20.5%	M +$N$5%	M +$N$2%
		M_{33} -22.5%	M +$N$5%
			M_{34} -23.5%

- $P(C_{21}) = P_M^4(\sum_{h=1}^{\infty} C_1^h P_{AM} P_M^{h-1}))P_{TOP} = P_M^4 P_{AM} P_{TOP}/(1-P_M)^2$
- $P(C_{22}) = P_M^4(\sum_{h=2}^{\infty} C_2^h P_{AM}^2 P_M^{h-2})P_{TOP} = P_M^4 P_{AM}^2 P_{TOP}/(1-P_M)^3$
- $P(C_{23}) = P_M^4(\sum_{h=3}^{\infty} C_3^h P_{AM}^3 P_M^{h-3})P_{TOP} = P_M^4 P_{AM}^3 P_{TOP}/(1-P_M)^4$
- $P(C_{24}) = P_M^4(\sum_{h=0}^{\infty}(P_M + P_{AM})^h)P_{TOP}$-$P(C_{15})$-$P(C_{21})$-$P(C_{22})$-$P(C_{23})$ = $P_M^4 P_{TOP}/(1-P_M-P_{AM})$-$P(C_{15})$-$P(C_{21})$-$P(C_{22})$-$P(C_{23})$

The probabilities of recruiting routes for form C3 are as follows:

- $P(C_{31}) = P_M^3 P_{AM}(\sum_{h=0}^{\infty} P_M^h)P_{TOP} = P_M^3 P_{AM} P_{TOP}/(1-P_M)$
- $P(C_{32}) = P_M^3 P_{AM}(\sum_{h=1}^{\infty} C_1^h P_{AM} P_M^{h-1})P_{TOP}$= $P_M^3 P_{AM}^2 P_{TOP}/(1-P_M)^2$
- $P(C_{33}) = P_M^3 P_{AM}(\sum_{h=2}^{\infty} C_2^h P_{AM}^2 P_M^{h-2})P_{TOP}$= $P_M^3 P_{AM}^3 P_{TOP}/(1-P_M)^3$
- $P(C_{34}) = P_M^3 P_{AM}(\sum_{h=0}^{\infty}(P_M + P_{AM})^h)P_{TOP}$ - $P(C_{31})$ - $P(C_{32})$ -$P(C_{33})$ = $P_M^3 P_{AM} P_{TOP}/(1-P_M-P_{AM})$- $P(C_{31})$ - $P(C_{32})$ -$P(C_{33})$

The probabilities of recruiting routes for form C4 are as follows:

- $P(C_{41}) = P_M^2 P_{AM}(\sum_{h=0}^{\infty} P_M^h)P_{TOP} = P_M^2 P_{AM} P_{TOP}/(1-P_M)$
- $P(C_{42}) = P_M^2 P_{AM}(\sum_{h=1}^{\infty} C_1^h P_{AM} P_M^{h-1})P_{TOP} = P_M^2 P_{AM}^2 P_{TOP}/(1-P_M)^2$
- $P(C_{43}) = P_M^2 P_{AM}(\sum_{h=2}^{\infty} C_2^h P_{AM}^2 P_M^{h-2})P_{TOP} = P_M^2 P_{AM}^3 P_{TOP}/(1-P_M)^3$
- $P(C_{44}) = P_M^2 P_{AM}(\sum_{h=0}^{\infty}(P_M + P_{AM})^h P_{TOP}$ - $P(C_{41})$ - $P(C_{42})$ - $P(C_{43}) = P_M^2 P_{AM} P_{TOP}/(1 - P_M - P_{AM})$ -$P(C_{41})$ - $P(C_{42})$ - $P(C_{43})$

The probabilities of recruiting routes for form C5 are as follows:

- $P(C_{51}) = P_M P_{AM}(\sum_{h=0}^{\infty} P_M^h)P_{TOP} = P_M P_{AM} P_{TOP}/(1-P_M)$
- $P(C_{52}) = P_M P_{AM}(\sum_{h=1}^{\infty} C_1^h P_{AM} P_M^{h-1})P_{TOP} = P_M P_{AM}^2 P_{TOP}/(1-P_M)^2$
- $P(C_{53}) = P_M P_{AM}(\sum_{h=2}^{\infty} C_2^h P_{AM}^2 P_M^{h-2})P_{TOP} = P_M P_{AM}^3 P_{TOP}/(1-P_M)^3$
- $P(C_{54}) = P_M P_{AM}(\sum_{h=0}^{\infty}(P_M + P_{AM})^h P_{TOP}$ - $P(C_{51})$ - $P(C_{52})$ - $P(C_{53}) = P_M P_{AM} P_{TOP}/(1 - P_M - P_{AM})$ - $P(C_{51})$ - $P(C_{52})$ - $P(C_{53})$

The probabilities of recruiting routes for form C6 are as follows:

- $P(C_{61}) = P_{AM}(\sum_{h=0}^{\infty} P_M^h)P_{TOP} = P_{AM} P_{TOP}/(1-P_M)$
- $P(C_{62}) = P_{AM}(\sum_{h=1}^{\infty} C_1^h P_{AM} P_M^{h-1})P_{TOP} = P_{AM}^2 P_{TOP}/(1-P_M)^2$
- $P(C_{63}) = P_{AM}(\sum_{h=2}^{\infty} C_2^h P_{AM}^2 P_M^{h-2})P_{TOP} = P_{AM}^3 P_{TOP}/(1-P_M)^3$
- $P(C_{64}) = P_{AM}(\sum_{h=0}^{\infty}(P_M + P_{AM})^h)P_{TOP}$ - $P(C_{61})$ - $P(C_{62})$ - $P(C_{63}) = P_{AM} P_{TOP}/(1-P_M-P_{AM})$ - $P(C_{61})$ - $P(C_{62})$ - $P(C_{63})$

The probabilities of recruiting routes for form M1 are as follows:

- $P(M_{11}) = P_{TOP}$
- $P(M_{12}) = P_M P_{TOP}$
- $P(M_{13}) = P_M^2 P_{TOP}$
- $P(M_{14}) = P_M^3(\sum_{h=0}^{\infty} P_M^h)P_{TOP} = P_M^3 P_{TOP}/(1-P_M)$

The probabilities of recruiting routes for form M2 are as follows:

- $P(M_{21}) = P_M P_{AM}(\sum_{h=0}^{\infty} P_M^h)P_{TOP} = P_M P_{AM} P_{TOP}/(1-P_M)$
- $P(M_{22}) = P_M P_{AM}(\sum_{h=1}^{\infty} C_1^h P_{AM} P_M^{h-1})P_{TOP} = P_M P_{AM}^2 P_{TOP}/(1-P_M)^2$
- $P(M_{23}) = P_M P_{AM}(\sum_{h=2}^{\infty} C_2^h P_{AM}^2 P_M^{h-2})P_{TOP} = P_M P_{AM}^3 P_{TOP}/(1-P_M)^3$

Table 10: The commission of recruiting routes for form M4

TOP +T0.5%	TOP +T0.5%	TOP +T0.5%	TOP +T0.5%
M (∞)	M (∞)	M (∞)	M (∞)
AM +$G$15% -$N$8%	AM +$N$5%	AM +$N$2%	AM +$N$1%
M +$N$1%	AM +$G$15% -$N$8%	AM +$N$5%	AM +$N$2%
M +$N$2%	M +$N$1%	AM +$G$15% -$N$8%	AM +$N$5%
M +$N$5%	M +$N$2%	M +$N$1%	AM +$G$15% -$N$8%
M_{41} -15.5%	M +$N$5%	M +$N$2%	M +$N$1%
	M_{42} -20.5%	M +$N$5%	M +$N$2%
		M_{43} -22.5%	M +$N$5%
			M_{44} -23.5%

- $P(M_{24}) = P_M P_{AM}(\sum_{h=0}^{\infty}(P_M + P_{AM})^h)P_{TOP}$ - $P(M_{21})$ - $P(M_{22})$ - $P(M_{23}) = P_M P_{AM} P_{TOP}/(1 - P_M - P_{AM})$ - $P(M_{21})$ - $P(M_{22})$ - $P(M_{23})$

The probabilities of recruiting routes for form M3 are as follows:

- $P(M_{31}) = P_M^2 P_{AM}(\sum_{h=0}^{\infty} P_M^h)P_{TOP} = P_M^2 P_{AM} P_{TOP}/(1-P_M)$
- $P(M_{32}) = P_M^2 P_{AM}(\sum_{h=1}^{\infty} C_1^h P_{AM} P_M^{h-1})P_{TOP} = P_M^2 P_{AM}^2 P_{TOP}/(1-P_M)^2$
- $P(M_{33}) = P_M^2 P_{AM}(\sum_{h=2}^{\infty} C_2^h P_{AM}^2 P_M^{h-2})P_{TOP} = P_M^2 P_{AM}^3 P_{TOP}/(1-P_M)^3$
- $P(M_{34}) = P_M^2 P_{AM}(\sum_{h=0}^{\infty}(P_M + P_{AM})^h)P_{TOP}$ - $P(M_{31})$ - $P(M_{32})$ - $P(M_{33}) = P_M^2 P_{AM} P_{TOP}/(1 - P_M - P_{AM})$ - $P(M_{31})$ - $P(M_{32})$ - $P(M_{33})$

The probabilities of recruiting routes for form M4 are as follows:

- $P(M_{41}) = P_M^3(\sum_{h=1}^{\infty} C_1^h P_{AM} P_M^{h-1})P_{TOP} = P_M^3 P_{AM} P_{TOP}/(1-P_M)^2$
- $P(M_{42}) = P_M^3(\sum_{h=2}^{\infty} C_2^h P_{AM}^2 P_M^{h-2})P_{TOP} = P_M^3 P_{AM}^2 P_{TOP}/(1-P_M)^3$
- $P(M_{43}) = P_M^3(\sum_{h=3}^{\infty} C_3^h P_{AM}^3 P_M^{h-3})P_{TOP} = P_M^3 P_{AM}^3 P_{TOP}/(1-P_M)^4$
- $P(M_{44}) = P_M^3(\sum_{h=0}^{\infty}(P_M + P_{AM})^h)P_{TOP}$ - $P(M_{14})$ -$P(M_{41})$ -$P(M_{42})$ -$P(M_{43}) = P_M^3 P_{TOP}/(1 - P_M - P_{AM})$ - $P(M_{14})$ -$P(M_{41})$ -$P(M_{42})$ -$P(M_{43})$

The probabilities of recruiting routes for form M5 are as follows:

- $P(M_{51}) = P_{AM}(\sum_{h=0}^{\infty} P_M^h)P_{TOP} = P_{AM} P_{TOP}/(1-P_M)$

Table 11: The commission of recruiting routes for form M5

TOP +T0.5%	TOP +T0.5%	TOP +T0.5%	TOP +T0.5%
M (∞)	M (∞)	M (∞)	M (∞)
AM +G15%	AM +N5%	AM +N2%	AM +N1%
M_{51} -15.5%	M (∞)	M (∞)	M (∞)
	AM +G15%	AM +N5%	AM +N2%
	M_{52} -20.5%	M (∞)	M (∞)
		AM +G15%	AM +N5%
		M_{53} -22.5%	M (∞)
			AM +G15%
			M_{54} -23.5%

Table 12: The commission of recruiting routes for form AM

TOP +T0.5%	TOP +T0.5%	TOP +T0.5%	TOP +T0.5%
M (∞)	M (∞)	M (∞)	M (∞)
AM_1 -0.5%	AM +N5%	AM +N2%	AM +N1%
	M (∞)	M (∞)	M (∞)
	AM_2 -5.5%	AM +N5%	AM +N2%
		M (∞)	M (∞)
		AM_3 -7.5%	AM +N5%
			M (∞)
			AM_4 -8.5%

- $P(M_{52}) = P_{AM}(\sum_{h=1}^{\infty} C_1^h P_{AM} P_M^{h-1}) P_{TOP} = P_{AM}^2 P_{TOP}/(1-P_M)^2$
- $P(M_{53}) = P_{AM}(\sum_{h=2}^{\infty} C_2^h P_{AM}^2 P_M^{h-2}) P_{TOP} = P_{AM}^3 P_{TOP}/(1-P_M)^3$
- $P(M_{54}) = P_{AM}(\sum_{h=0}^{\infty}(P_M + P_{AM})^h) P_{TOP}$ - $P(M_{51})$ -$P(M_{52})$ -$P(M_{53})$ = $P_{AM}P_{TOP}/(1 - P_M - P_{AM})$ -$P(M_{51})$ -$P(M_{52})$ -$P(M_{53})$

The probabilities of recruiting routes for form AM are as follows:

- $P(AM_1) = (\sum_{h=0}^{\infty} P_M^h) P_{TOP}$ = $P_{TOP}/(1-P_M)$
- $P(AM_2) = (\sum_{h=1}^{\infty} C_1^h P_{AM} P_M^{h-1}) P_{TOP} = P_{AM}P_{TOP}/(1-P_M)^2$
- $P(AM_3) = (\sum_{h=2}^{\infty} C_2^h P_{AM}^2 P_M^{h-2}) P_{TOP} = P_{AM}^2 P_{TOP}/(1-P_M)^3$
- $P(AM_4)$ = 1- $P(AM_1)$ -$P(AM_2)$ -$P(AM_3)$

Table 13: The mean, variance and standard deviation of commission should an employee offers to employees multilevel above

Example	1.	2.
P_M	0.7	0.94
P_{AM}	0.2	0.05
P_{TOP}	0.1	0.01
μ_C	23.64%	29.20%
μ_M	15.12%	19.37%
μ_{AM}	5.02%	6.65%
σ_C^2	0.010698	0.004144
σ_M^2	0.003864	0.002508
σ_{AM}^2	0.001136	0.000856
σ_C	10.34%	6.44%
σ_M	6.22%	5.01%
σ_{AM}	3.37%	2.93%

6 Mean, Variance and Standard Deviation

Consider the first example $P_M = 0.7$, $P_{AM} = 0.2$, $P_{TOP} = 0.1$, and the second example $P_M = 0.94$, $P_{AM} = 0.05$, $P_{TOP} = 0.01$, in Table 13.

The mean value of the maximum cost in the first example is (100%-23.64%)*R_C+ (100%-15.12%)*R_M+ (100%-5.02%)*R_{AM}.

The mean value of the maximum cost in the second example is (100%-29.20%)*R_C+ (100%-19.37%)*R_M+ (100%-6.65%)*R_{AM}.

The R_C, R_M, and R_{AM} are not identical in the two examples. However, the following conclusions can be drawn:

1. Employees in lower stages must pay more commission to multilevel above employees, particularly in a company with few top employees.
2. The commission is as high as 33.5% in Tables 2 to 6; therefore, in this case, the cost is a maximum of 100% to -33.5%=66.5%. If the actual purchasing price is also counted, the commission should be 33.5%/(1-0.16) =39.9%. In this case the cost is a maximum of 100% to subtract 39.9%=60.1%.

7 Conclusion

A number of researchers have argued that a good direct-selling company should allow employees to earn at least 75% of the commission, that is, should

control the product cost to at most 25%. However, perhaps they themselves may not be willing to pay 75% commission to other employees. So it's challenging to recruit adequate employees to earn 75% commission.

Locating a cheaper replacement for the low-cost direct selling product, such as a bottle of drinking water with maximum cost not greater than 10%, is relatively easy. Employees should individually judge whether the direct selling products are worth to purchasing and selling.

The simplified model proposed in this study can be extended to a comprehensive model for this company. Adopting this model enables the actual employees proportions in all the stages to be used. Additionally, more combinations of employee proportions in all the stages can be simulated, and the rules on upgrading and commission of other direct selling companies can be obtained. Then, different direct selling companies can be compared, and the possibly corrupt companies identified.

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