A Study on the Complexity of Multi-Enterprise Output Game in Supply Chain

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Abstract: Based on an analysis on a variety of game models in the supply chain, this paper proposes a multi-enterprise output game model under the circumstances of information asymmetry. After a study on Nash equilibrium, the paper analyzes the factors that cause chaos in production decision making among manufacturers and offers numerical simulation. The authors argue that under the circumstances of information asymmetry, the differences in productivity adjustment factor exert great impact on the manufacturers, shown as bifurcation and chaos, whereas the distributors and retailers are scarily affected. The measures to keep the chaos among manufacturers are suggested.

Key-Words: - supply chain; output game; bifurcation; Lyapunov exponents; discrete dynamical system; complexity

1 Introduction

With the advent of economic globalization and knowledge economy, supply chain management is widely used in manufacturing management. It is based on the demand of the customers and the market that the manufacturers develop products, purchase raw materials which are then processed into finished goods and sold to customers. With the further division of labor, the enterprises in the supply chain are more and more specialized in certain subsections along the product life cycle. There is a constant and complicated game among enterprises for the allocation of productivity and profit.

Yingxue Zhao et.al [1] took a cooperative game approach to consider the coordination issue in a manufacturer-retailer supply chain using option contracts. They developed an option contract model using the wholesale price mechanism as a benchmark. Mingming Leng and An Zhu [2] investigated supply chain coordination with side-payment contracts. They discussed two criteria that a proper side-payment contract must satisfy, and introduced a decision-dependent transfer payment function and a constant transfer term. Mahesh Nagarajan et.al [3] describes the construction of the set of feasible outcomes in commonly seen supply chain models, and uses cooperative bargaining models to find allocations of the profit pie between supply chain partners. Kirsi Korhonen et.al [4] demonstrate with a special case the use of role game as a tool to increase process understanding and communication skills in a kick-off workshop of a supply chain improvement program.

The researches mentioned above focus on the multi-players along the supply chain with a perspective from game theory. Interlinked to form a chain, the enterprises along the supply chain are interdependent and mutually restrained. When making decisions, these enterprises are not fully rational. They need to take into consideration multiple factors such as their cost, profit, the change in market demand, decisions of their competitors and partners. Therefore, conventional decision making modeling cannot meet the challenge raised by a complex supply chain. In the next section, we construct a discrete dynamical model of three-level supply chain under the circumstances of information asymmetry and analyze its dynamic behavior in light of nonlinear dynamics.

2 Model Constructions and Analysis

2.1 Assumptions

This model is based on these following assumptions.

Assumption 1: Different importance is assigned to the three traditional players along the supply chain, with the manufacturers playing a dominant role.

Assumption 2: At each of the three levels along the supply chain, there is only one enterprise that enjoys monopoly.

Assumption 3: In light of the complexity in reality, enterprises have respective nonlinear cost functions and nonlinear inverse demand functions.

Assumption 4: Enterprises always make the optimal output decision for the maximum margin profit in every period.

Assumption 5: Production decision making at each level is based on the demand of the enterprises at the lower level.

Assumption 6: For the convenience of the study, make the output volume of the distributors equal to their order volume and the output volume of the retailers equal to their order volume.

For the convenience of study, we choose three traditional players along the supply chain, the manufacturers, the distributors and the retailers, to represent respectively the upstream, the midstream and the downstream.

2.2. Nomenclature and Model Construction

The following is a list of notations that will be used throughout the paper.

$q_{i,t}$ is the production decision making of enterprise i in period t.

$$P_{i,t} = \alpha_i + \beta_i q_{i,t} - \gamma_i q_{i,t}^2 \tag{1}$$

$P_{i,t}$ is the nonlinear inverse demand function for enterprise i in period t.

$$C_{i,t} = a_i + b_i q_{i,t} + c_i q_{i,t}^2 \tag{2}$$

$C_{i,t}$ is the cost function for enterprise i in period t.

$$\pi_{i,t} = P_{i,t} q_{i,t} - C_{i,t} \tag{3}$$

$\pi_{i,t}$ is the profit of enterprise i in period t.

Due to the bounded rationality and information asymmetry among the enterprises along the supply chain, when it comes to production decision making, the enterprises tend to increase the output until the maximum margin profit is attained. Therefore,

$$\partial \pi_{i,t} / \partial q_{i,t} = -3\gamma_i q_{i,t}^2 + 2(\beta_i - c_i) q_{i,t} + \alpha_i - b_i \tag{4}$$

Since the decision making process is long, repetitive and dynamic, it is characterized by adaptability and long-term memory effect. In most cases, it is rationally bounded. When the enterprises realize that the results achieved in period t is satisfactory, they will follow the same strategy in period t+1. The aim of the enterprises is the profit maximization. Marginal output is one of the strategies that they adopt in the game. If the marginal output in period t is positive, then they will continue their output adjustment strategy in period t+1. The model can be constructed as follow:

$$q_{i,t+1} = q_{i,t} + k_i q_{i,t} \partial \pi_{i,t} / \partial q_{i,t} \tag{5}$$

where k_i is the output adjustment coefficient for enterprise i .

Then, the dynamic adjustment of the output of the upstream enterprises—the distributors—can be written as follow:

$$q_{1,t+1} = q_{1,t} + k_1 q_{1,t}(-3\gamma_1 q_{1,t}^2 + 2(\beta_1 - c_1) q_{1,t} + \alpha_1 - b_1) \tag{6}$$

Likewise, the dynamic adjustment of the output of enterprise i can be written as follow:

$$q_{i,t+1} = q_{i,t} + k_i q_{i,t}(-3\gamma_i q_{i,t}^2 + 2(\beta_i - c_i) q_{i,t} + \alpha_i - b_i) \tag{7}$$

Hence, the output game model can be represented by an n-dimensional nonlinear map.

$$\begin{cases} x_1' = x_1 + k_1 x_1[-3\gamma_1 x_1^2 + 2(\beta_1 - c_1)x_1 + \alpha_1 - b_1] \\ x_2' = x_2 + k_2 x_2[-3\gamma_2 x_2^2 + 2(\beta_2 - c_2)x_2 + \alpha_2 - b_2] \\ \vdots \\ x_i' = x_i + k_i x_i[-3\gamma_i x_i^2 + 2(\beta_i - c_i)x_i + \alpha_i - b_i] \\ \vdots \\ x_n' = x_n + k_n x_n[-3\gamma_n x_n^2 + 2(\beta_n - c_n)x_n + \alpha_n - b_n] \end{cases} \tag{8}$$

We can interpret this multi-dimensional discrete dynamical system through a study on the first map. Assume the retailers make adjustment on their ordering strategies in response to the changes in market demand, the distributors to the changes in the retailers' orders and the manufacturers to the changes in the distributors' orders. Then a nonlinear dynamic model is constructed as follows:

Let $q_{1,t} = x$, $q_{2,t} = y$, $q_{3,t} = z$, $q_{1,t+1} = x'$, $q_{2,t+1} = y'$, $q_{3,t+1} = z'$. Then:

$$\begin{cases} x' = x + k_1 y[-3\gamma_1 x^2 + 2(\beta_1 - c_1)x + \alpha_1 - b_1] \\ y' = y + k_2 z[-3\gamma_2 y^2 + 2(\beta_2 - c_2)y + \alpha_2 - b_2] \\ z' = z + k_3 z[-3\gamma_3 z^2 + 2(\beta_3 - c_3)z + \alpha_3 - b_3] \end{cases} \tag{9}$$

2.3 Model Analysis

The fixed points in the model satisfy the following algebraic equations:

$$\begin{cases} k_1 x_1[-3\gamma_1 x_1^{\ 2} + 2(\beta_1 - c_1)x_1 + \alpha_1 - b_1] = 0 \\ k_2 x_2[-3\gamma_2 x_2^{\ 2} + 2(\beta_2 - c_2)x_2 + \alpha_2 - b_2] = 0 \\ \vdots \\ k_i x_i[-3\gamma_i x_i^{\ 2} + 2(\beta_i - c_i)x_i + \alpha_i - b_i] = 0 \\ \vdots \\ k_n x_n[-3\gamma_n x_n^{\ 2} + 2(\beta_n - c_n)x_n + \alpha_n - b_n] = 0 \end{cases} \tag{10}$$

Note that the solution of the algebraic equations is independent of parameters k_1, k_2, $\cdots$, k_n. For models in economics, only non-negative equilibrium solution makes sense. By simple computation of the above algebraic system, an equilibrium is found as follows:

$$E^*(x_1^*, x_2^*, \cdots, x_n^*) = (\frac{\beta_1 - c_1 + \Omega_1}{3\gamma_1}, \frac{\beta_2 - c_2 + \Omega_2}{3\gamma_2}, \cdots \frac{\beta_n - c_n + \Omega_n}{3\gamma_n}),$$

Where

$$\Omega_1 = \sqrt{(\beta_1 - c_1)^2 + 3\gamma_1(\alpha_1 - b_1)}$$

$$\Omega_2 = \sqrt{(\beta_2 - c_2)^2 + 3\gamma_2(\alpha_2 - b_2)}$$

$$\Omega_n = \sqrt{(\beta_n - c_n)^2 + 3\gamma_n(\alpha_n - b_n)}.$$

The Jacobian matrix at Nash equilibrium can be represented by the following form:

$$J(E^*) = \begin{bmatrix} 1 + k_1\omega_1 & 0 & \cdots & 0 & 0 \\ 0 & 1 + k_2\omega_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & 1 + k_n\omega_n \end{bmatrix}$$

where $\omega_i = -9\gamma_i x^2 + 4(\beta_i - c_i)x_i + \alpha_i - b_i$.

The above is a diagonal matrix and its eigenvalues can be expressed as follows:

$\lambda_1 = 1 - k_1\omega_1$, $\lambda_2 = 1 - k_2\omega_2$, $\dots$, $\lambda_n = 1 - k_n\omega_n$

Take the three traditional players into consideration, and the algebraic equations can be written as follows:

$$\begin{cases} k_1 y[-3\gamma_1 x^2 + 2(\beta_1 - c_1)x + \alpha_1 - b_1] = 0 \\ k_2 z[-3\gamma_2 y^2 + 2(\beta_2 - c_2)y + \alpha_2 - b_2] = 0 \\ k_3 z[-3\gamma_3 z^2 + 2(\beta_3 - c_3)z + \alpha_3 - b_3] = 0 \end{cases} \tag{11}$$

It is obvious that the equilibrium does not depend on the parameters k_1, k_2, k_3. When the output is negative, the system is meaningless. $p(1.2585, 1.3067, 1.3557)$ is the Nash equilibrium of the system. The Jacobian matrix at p has the following form:

$$J = \begin{bmatrix} 1 - 9.6056k_1 & 0.0002k_1 & 0 \\ 0 & 1 - 10.0867k_2 & 0.00028k_2 \\ 0 & 0 & 1 - 10.2145k_3 \end{bmatrix} \tag{12}$$

The above can be regarded as a diagonal matrix (the values of $0.0002k_i$ and $0.00028k_2$ can be overlooked as they are trivial in comparison to the inaccuracies caused by smaller number of iterations) and its Eigen values can be expressed as follow:

$\lambda_1 = 1 - 9.6056k_1$. If $k_1 = 0.2082$, then $\lambda_1 = -1$. Therefore, period doubling bifurcation can be found at $k_1 = 0.2082$ in the system.

In the dynamic output game model, parameters $\alpha_i, \beta_i, \gamma_i, a_i, b_i, c_i$ are relatively fixed whereas the output adjustment coefficients k_1, k_2, k_3 are not. For the convenience of the study, make the parameters fixed as follows:

α_1 =5, β_1 =0.5, γ_1 =1, b_1 =0.5, c_1 =0.4, α_2 =5, β_2 =0.5, γ_2 =1, b_2 =0.4, c_2 =0.3, α_3 =5, β_3 =0.5, γ_3 =1, b_3 =0.3, c_3 =0.2.

Then the dynamic output game model can be written as:

$$\begin{cases} x' = x + k_1 y[-3x^2 + 0.2x + 4.5] \\ y' = y + k_2 z[-3y^2 + 0.4y + 4.6] \\ z' = z + k_3 z[-3z^2 + 0.6z + 4.7] \end{cases} \tag{13}$$

For a better understanding of the dynamics in the system, we conduct numerical simulations. Take into consideration one instance, that is coefficient k_1 is free and coefficient k_2, k_3 are fixed. When $k_2 = 0.03$, $k_3 = 0.02$, simulate the impact of the change in the manufacturers' output adjustment speed k_1 on the system's complexity change. We first illustrate the changes in output decision making of the three players by bifurcation diagrams. Then we study the relationship between the complexity of the system and k_1 by using Lyapunov exponents, attractors, sequence diagrams and sensitive dependence on initial conditions. The simulation results are interpreted with a perspective from supply chain.

Bifurcation Fig.1 shows that the manufacturers' output is stable with $k_1 \in (0, 0.2082)$. With the increase in k_1, when the first bifurcation occurs at $k_1 = 0.2082$ and the second at $k_1 = 0.2661$, the third at $k_1 = 0.2785$, …, chaos occurs. Bifurcation diagrams Fig.2 and Fig.3, however, show that the output decision making of the distributors and the retailers is stable. Therefore, it can be concluded that the greater the speed of output adjustment becomes, the faster the output decision making responds to the change in market demand, and the more likely that chaos will occur in the market. Complexity will be caused in the manufacturer's output and production management. In contrast, the distributors and the retailers are stable as they are not involved in production.

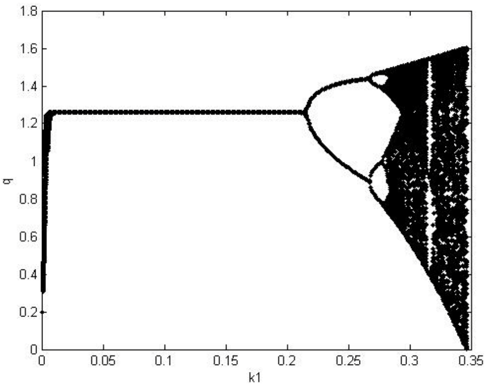

Fig.1 Output bifurcation diagram of manufacturer X

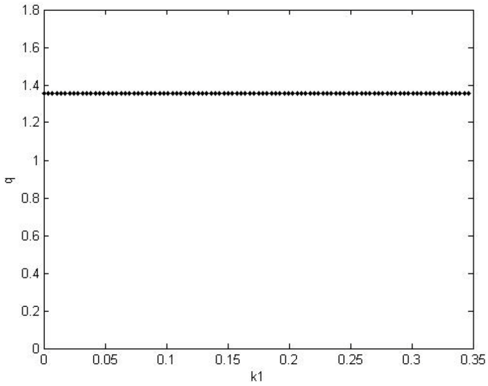

Fig.2 Output bifurcation diagram of distributor Y

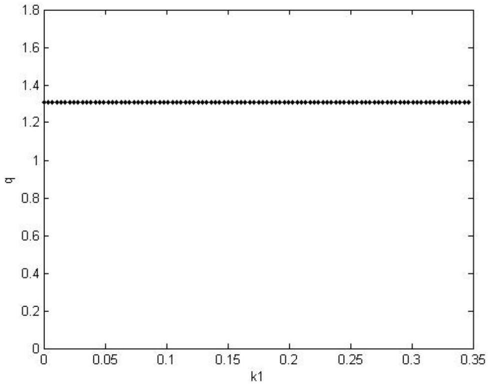

Fig.3 Output bifurcation diagram of retailer Z

The Lyapunov exponent is one of the most useful quantitative measures of chaos. A positive largest Lyapunov exponent indicates chaos.

According to Lyapunov exponent Fig.4, the system is stable when $k_1 \in (0, 0.2082)$. When $k_1 < 0.2785$, the system undergoes period doubling bifurcation and the oscillation is periodic. Therefore, the maximum Lyapunov exponent always has a value less than zero. It becomes equal to zero exactly at the bifurcation point. When $k_1 > 0.2785$, the majority Lyapunov exponents have a value greater than zero, indicating that the system is chaotic. The maximum Lyapunov exponent is less than zero only within a narrow range which corresponds to a periodic window in chaos.

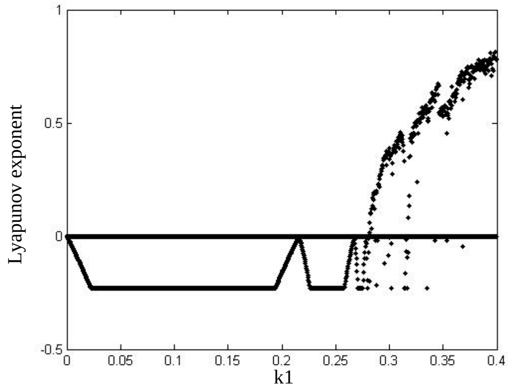

Fig.4 The first Lyapunov exponent

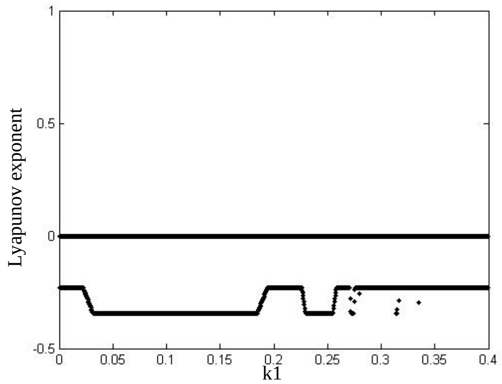

Fig.5 The second Lyapunov exponent

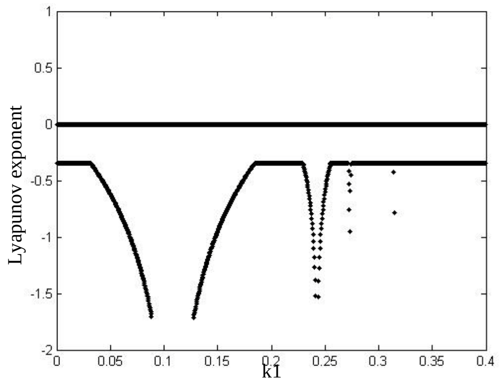

Fig.6 The third Lyapunov exponent

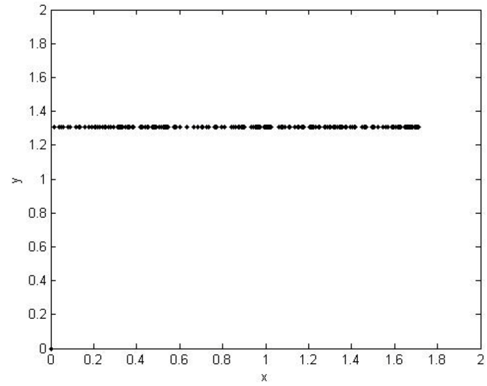

Fig.7 X and Y chaotic attractors

The chaotic attractors in Fig.7 and Fig.8 are peculiar in that they form a dashed line, which is caused by the fact that in the system only the manufacturers are in chaos, whereas the distributors and retailers are stable. Fig.9 shows the attractors of the distributors and retailers. They merge into a dot because the distributors and retailers are stable. Fig.10 shows the three-dimensional attractors of the manufacturers, the distributors and the retailers.

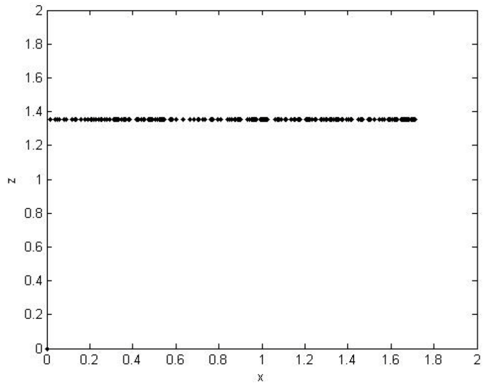

Fig.8 X and Y chaotic attractors

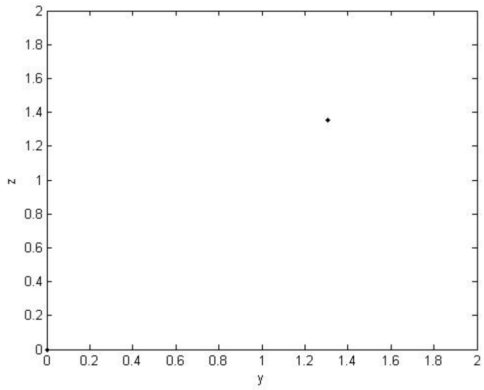

Fig.9 X and Y chaotic attractors

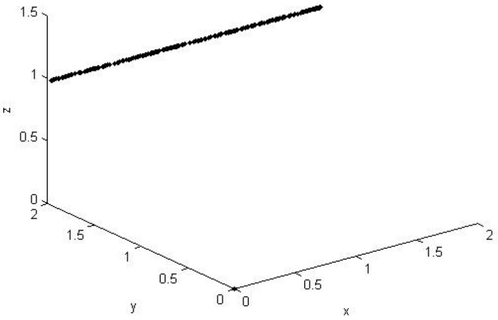

Fig.10 XYZ chaotic attractors

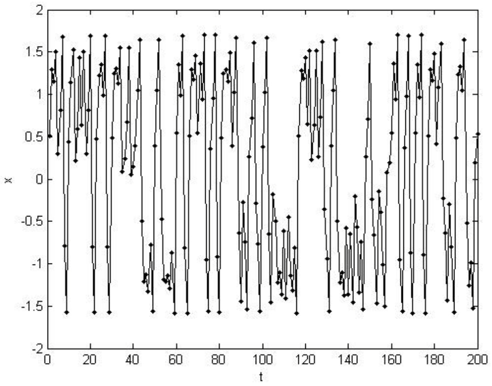

Fig.11 Sequence diagram of manufacturer X

Fig.11, Fig.12 and Fig.13 are the sequence diagrams of the three players in the system. These diagrams give proof to the analysis made above: the manufacturers are chaotic whereas the distributors and retailers are stable.

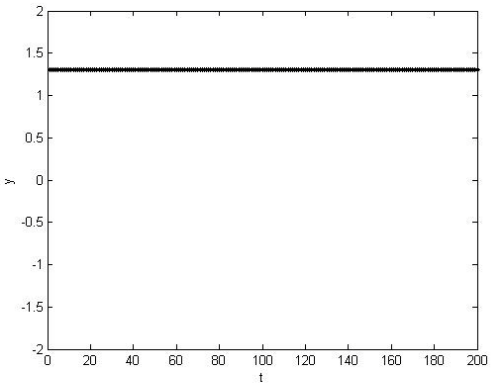

Fig.12 Sequence diagram of distributor Y

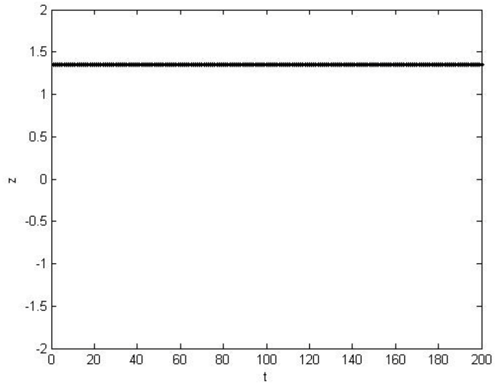

Fig.13 Sequence diagram of retailer Z

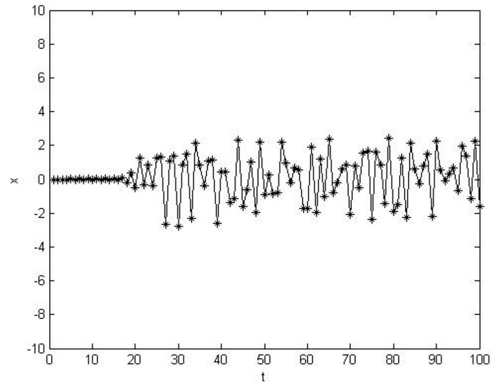

Fig.14 Sensitive dependence of manufacturer X

Fig.14, Fig.15 and Fig.16 show the change in the output of the manufacturers, the distributors and the retailers with time passing. As the chaotic motion is sensitively dependent on the initial conditions, trivial differences in the initial conditions will lead to the departure of two adjacent trajectories. Replace the initial conditions (0.2, 0.4, 0.6) with (0.2, 0.4, 0.6001), as shown in Fig.16. It is found that the difference at the initial stage is close to zero. With the passing of time, the differences in the manufacturers' output decision making increase after 20 iterations, leading the adjacent trajectories into different domains of attraction. In contrast, the distributors in Fig.15 and the retailers in Fig.16 tend to stabilize after 30 iterations. These further support the analysis made above.

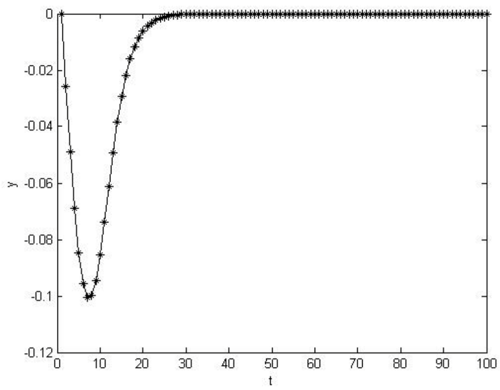

Fig.15 Sensitive dependence of distributor Y

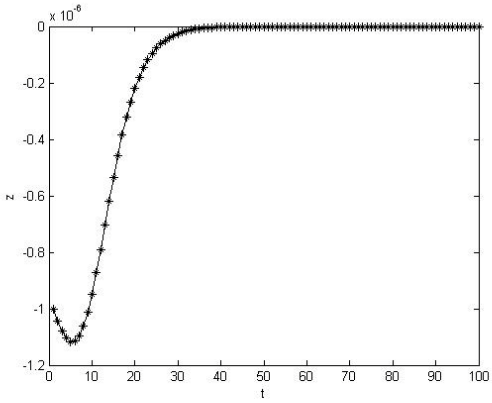

Fig.16 Sensitive dependence of retailer Z

Assume that the initial condition of the model is the even distribution of $[0,1]\times[0,1]\times[0,1]$, and every manufacturer can decide his output between 0 and max value. The output adjustment coefficient of these three manufacturers reaches a stable density state and is fixed at an equilibrium (0.5709, 0.6544, 0.7520), as is shown in Fig.17(a), after multiple games when it satisfies the condition k1=k2=k3=0.252. When the output adjustment coefficient satisfies the condition k1=k2=k3=0.254, the density distribution of these three manufacturers circulates between (0.9177, 0.8252, 0.8126) and (0.9128, 0.8506, 0.8516) after 2,000 games, which exhibits double period of density, as is shown in Fig.17(b). And when the output adjustment coefficient satisfies the condition k1=k2=k3=0.255, the output's density distribution is no longer stable, and exhibits quadruple period of density, as is shown in Fig.17 (c).

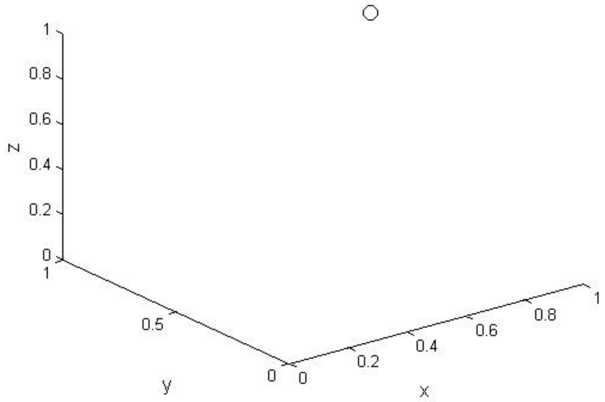

Fig.17 (a)

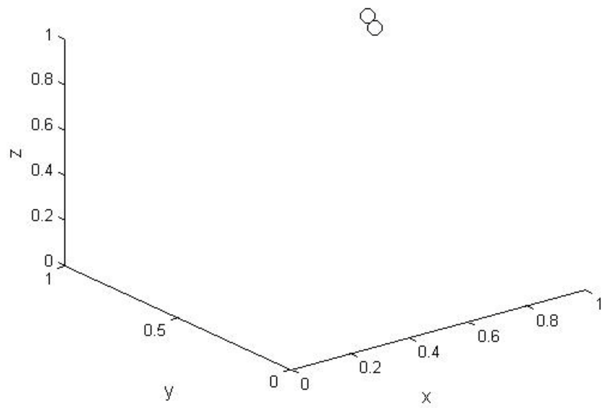

Fig.17 (b)

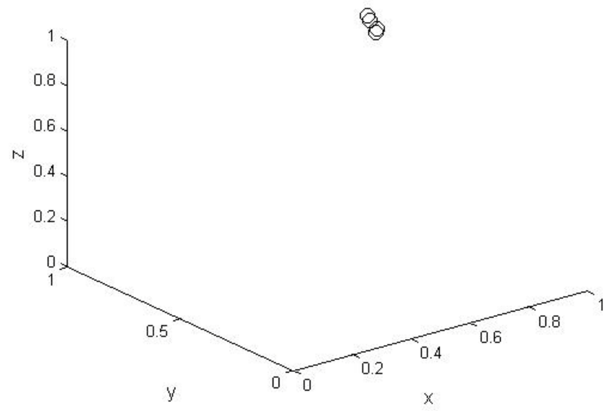

Fig.17 (c)

Fig.17 Output density distribution after 2,000 games when k_1=k_2=0.252(a), k_1=k_2= 0.254(b), k_1=k_2=0.255(c), k_1=k_2=0.256(d),
k_1=k_2=0.257(e), k_1=k_2=0.258(f)

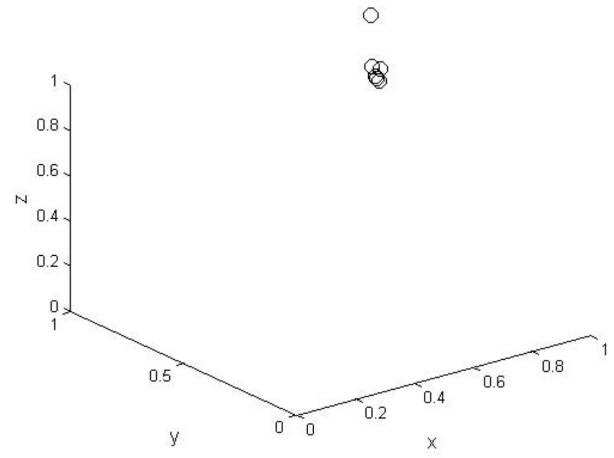

Fig. 17 (d)

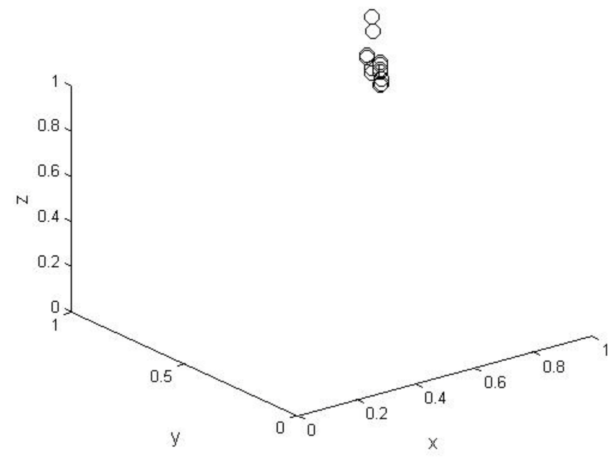

Fig.17 (e)

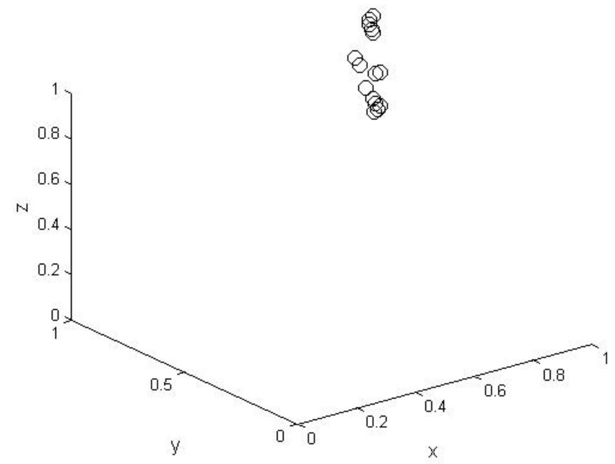

Fig.17 (f)

When the output adjustment coefficient satisfies the condition k1=k2=k3=0.256, the output's density distribution exhibits octuple period of density, as is shown in Fig.17(d). When the output adjustment coefficient satisfies the condition k1=k2=k3=0.257, the output's density distribution exhibits sixteen period of density, as is shown in Fig.17 (e). As the output adjustment coefficient increases, the system is in chaos, as shown in Fig.17(f).

If we continue increasing the adjustment coefficient of the output till k1=k2=k3=0.246, then we can find the octo-periodic phenomenon, which means that we can find eight Nash equilibrium solution, and according to the conclusion of Li-York, at the third period of this phenomenon, the chaos appears and all of the system is in chaos. And at this time, even a tiny change of the adjustment coefficient of the output can cause a huge difference of the distribution of the density. And with the increasing of the adjustment coefficient of the output to k1=k2=k3=0.248, and k1=k2=k3=0.249, the density's sixteen-periodic phenomenon and sixty-four-periodic phenomenon appear and the whole system is in the condition of chaos.

The density distribution diagrams show that when the decision making is stable, the manufacturers fix the output at 0.927, and the distributors fix the ordering volume at 0.8467, and the retailers at 0.8389. The output of the manufacturers is always larger than the ordering volume of the distributors which is larger than that of the retailers. This is result of reverse logistics from retailers to distributors and then to manufacturers.

2.4 Chaos control

Chaos control aims at altering the chaotic motions in non-linear dynamical systems to display periodic dynamics. Methods of chaos control include OGY method, The OGY method refers to the control method of Ott, Grebogi and Yorke. Put forward in 1990 by American physicists Edward Ott, Celso Grebogi and James A. Yorke, it stabilizes a hyperbolic periodic orbit by making small perturbations for a system parameter. Explanation on the OGY method has been added to the new version, variable structure control and impulse control, and so on. The ground that these methods share in common is that the Lyapunov exponent is adjusted from positive to negative so that stability is achieved in the originally unstable system [10-11]. In other words, for the purpose of chaos control, control signal is fed to the system for bifurcation control so that the chaos is eliminated or delayed.

Adjust the original system so that:

$$\begin{cases} q_1(t+1) = X\left(q_1(t), q_2(t), q_3(t)\right) \\ q_2(t+1) = Y\left(q_1(t), q_2(t), q_3(t)\right) \\ q_3(t+1) = Z\left(q_1(t), q_2(t), q_3(t)\right) \end{cases}$$

(14)

Apply the following strategies of parameter adjustment and state feedback control：

$$\begin{cases} q_1(t+m) = \mu X^{(m)}\left(q_1(t), q_2(t), q_3(t)\right) + (1-\mu)q_1(t) \\ q_2(t+m) = \mu Y^{(m)}\left(q_1(t), q_2(t), q_3(t)\right) + (1-\mu)q_2(t) \\ q_3(t+m) = \mu Z^{(m)}\left(q_1(t), q_2(t), q_3(t)\right) + (1-\mu)q_3(t) \end{cases} \quad (15)$$

Feed control signal μ to the system, then the response of enterprise at level i satisfies the condition $\mu_i \in [0,1]$. Assume all the enterprises share the same control rule, then the output decision making function of enterprise at level i can be written as:

$$x_i' = \mu_i(x_i + k_i x_i(-3\gamma_i x_i^2 + 2(\beta_i - c_i)x_i + \alpha_i - b_i)) + (1-\mu_i)x_i \quad (16)$$

Therefore, the output game model of the system-controlled enterprise can be written as

$$\begin{cases} x_1' = \mu_1(x_1 + k_1 x_1(-3\gamma_1 x_1^2 + 2(\beta_1 - c_1)x_1 \\ +\alpha_1 - b_1)) + (1-\mu_1)x_1 \\ x_2' = \mu_2(x_2 + k_2 x_2(-3\gamma_2 x_2^2 + 2(\beta_2 - c_2)x_2 \\ +\alpha_2 - b_2)) + (1-\mu_2)x_2 \\ \vdots \\ x_i' = \mu_i(x_i + k_i x_i(-3\gamma_i x_i^2 + 2(\beta_i - c_i)x_i \\ +\alpha_i - b_i)) + (1-\mu_i)x_i \\ \vdots \\ x_n' = \mu_n(x_n + k_n x_n(-3\gamma_n x_n^2 + 2(\beta_n - c_n)x_n \\ +\alpha_n - b_n)) + (1-\mu_n)x_n \end{cases} \quad (17)$$

The analysis on the equilibrium point of the model is not elaborated here, because it follows the same train of thought. As the solution for the mathematical formula group is independent of parameter μ, the Nash equilibrium solution is fixed.

When μ =1, the system degenerates into the original system. Select the appropriate adjustment parameter μ so that the equilibrium will remain stable in a scale larger than the original system and the bifurcation will be delayed.

Feed in the parameter mentioned above, and the controlled model can be written as

$$\begin{cases} x' = \mu(x + k_1 y(-3x^2 + 0.2x + 4.5)) + (1-\mu)x \\ y' = \mu(y + k_2 y(-3y^2 + 0.4y + 4.6)) + (1-\mu)y \\ z' = \mu(z + k_3 z(-3z^2 + 0.6z + 4.7)) + (1-\mu)z \end{cases} \quad (18)$$

Numerical simulations and analysis on the model are provided below by using bifurcation diagrams and Lyapunov exponents. Chaos control parameter μ is adjusted for the study of the control effect on the chaotic output decision making of the manufacturers. As the distributors' ordering decision making is stable, no control is exercised on it.

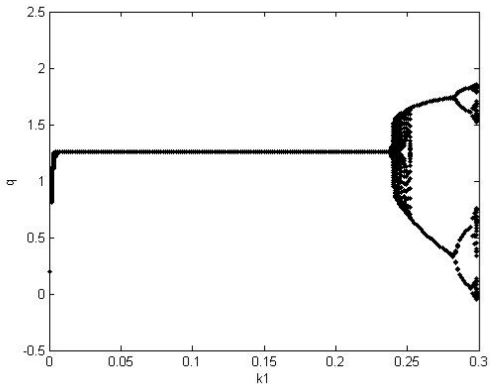

Fig.18 Bifurcation diagram of manufacturer x when μ=0．9

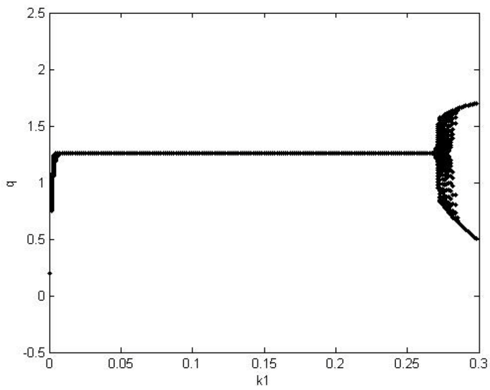

Fig.19 Bifurcation diagram of manufacturer x when μ=0．8

Fig.18 is the output bifurcation diagrams of manufacturer x. Make the controlling parameter μ=0.9, then the first bifurcation occurs in the system when k_1 =0.2385, the second when

k_1 =0.2835, and the third when k_1 =0.2955,…, till chaos occurs. The stable area in Fig. 18 is larger than that in Fig 1, into which the controlling parameter μ is not fed. Make the controlling parameter μ=0.8, then the first bifurcation occurs in the system when k_1 =0.2685 (Fig. 19). The bifurcation diagram of the system shows a dashed line when the controlling parameter μ=0.7, which indicates that the system is stable (Fig. 20).

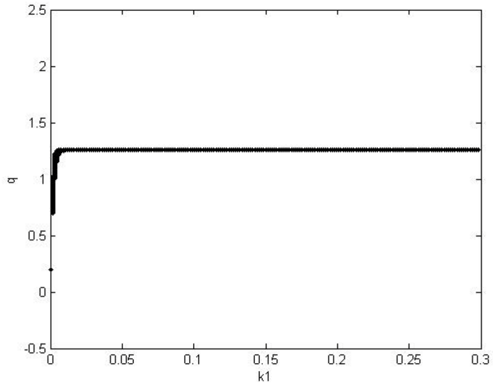

Fig.20 Bifurcation diagram of manufacturer x when μ=0. 7

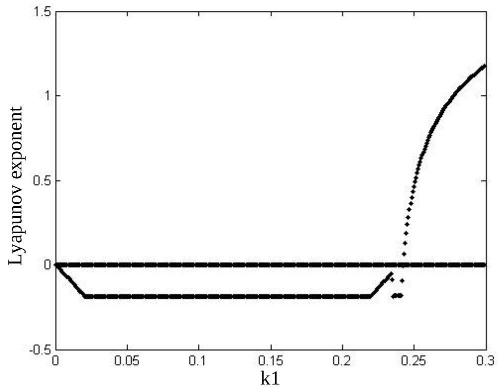

Fig.21 The first Lyapunov exponent When μ=0.9

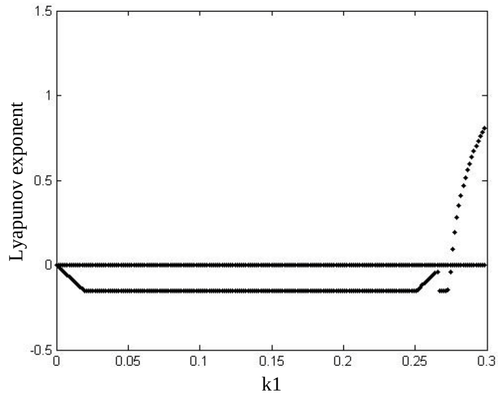

Fig.22 The first Lyapunov exponent When μ=0.8

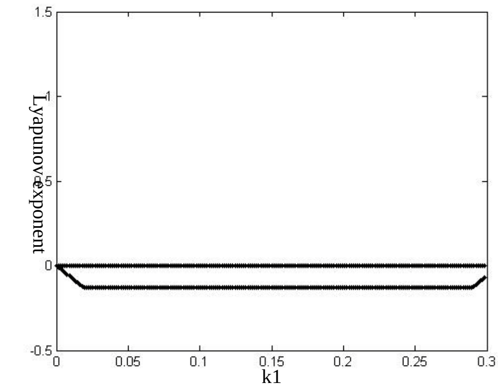

Fig.23 The first Lyapunov exponent When μ=0.7

Adjust parameter μ, then the Lyapunov spectrum of manufacturer x and distributor y changes in the same way as the bifurcation diagram of the system shows. As is shown in Fig.21, make the controlling parameter μ=0.9, the system is stable when $k_1 \in (0, 0.2385)$. When $k_1 < 0.243$, the system is in the process of period-doubling bifurcation. When $k_1 > 0.243$, the system is chaotic. In contrast to Fig. 4, the area where the Lyapunov exponent is negative is enlarged, indicating that the control effect is achieved. When the control parameter μ=0.8 (Fig.22) the system is chaotic when $k_1 > 0.243$; When the control parameter μ=0.7 (Fig.23), the maximum Lyapunov exponent of the system is less than zero. And the smaller μ is, the more stable the system is.

Based on the analysis above, three typical control solutions are worked out:

(1) Make μ=0.9, the system is chaotic.
(2) Make μ=0.8, the system is in bifurcation.
(3) Make μ=0.7, the system is stable.

Make controlling parameter μ=0.9, the system is chaotic, but the stable area is larger and there is chaos lag. In this case, the effect is noticeable and it is easy to control, but it is not suitable for long-term control. Make the controlling parameter μ=0.8, the chaos is brought under control and the system is in bifurcation. In this case, the maximum Lyapunov exponent is zero, indicating the emergence of chaotic periodical window. This solution will lead to instability and the system might be brought back to chaos due to the interaction between some controlling factors. Make controlling parameter μ=0.7, chaos in the system is brought under complete control. The maximum Lyapunov exponent is less than zero, indicating the system is in stability. However, it is difficult to put this solution into reality and the cost involved is high.

3 Conclusion

This paper focuses on the repeated output decision making game of the manufacturers, the distributors and the retailers in response to demand fluctuation. The rules of the output decision making games are studied by means of system stability analysis and Lyapunov exponents. In the three-level supply chain, the demand fluctuation among the retailers will finally cause chaos in manufacturers' output decision making. This phenomenon is called the bullwhip effect. It has negative impact on the manufacturers in terms of production and sales. This paper proposes models and analysis that will lend light to a set of measures in tackling the negative impact arising from the chaos among the manufacturers. For example, the government can limit the manufacturers' output adjustment speed within range necessary through taxation and fiscal policies.

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